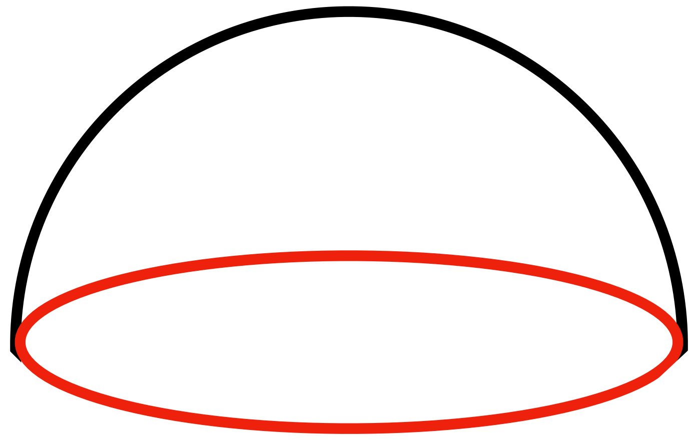
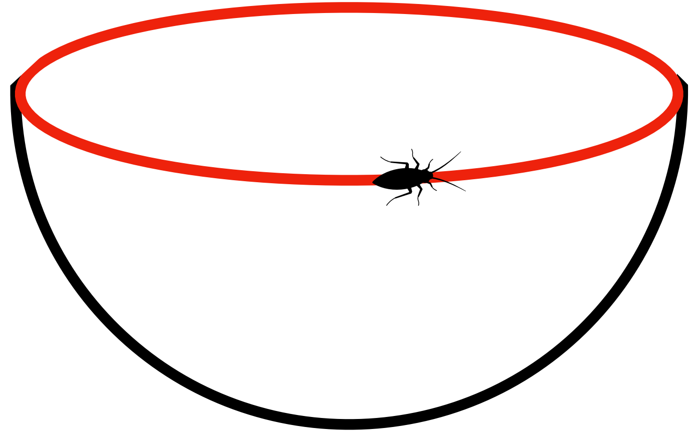
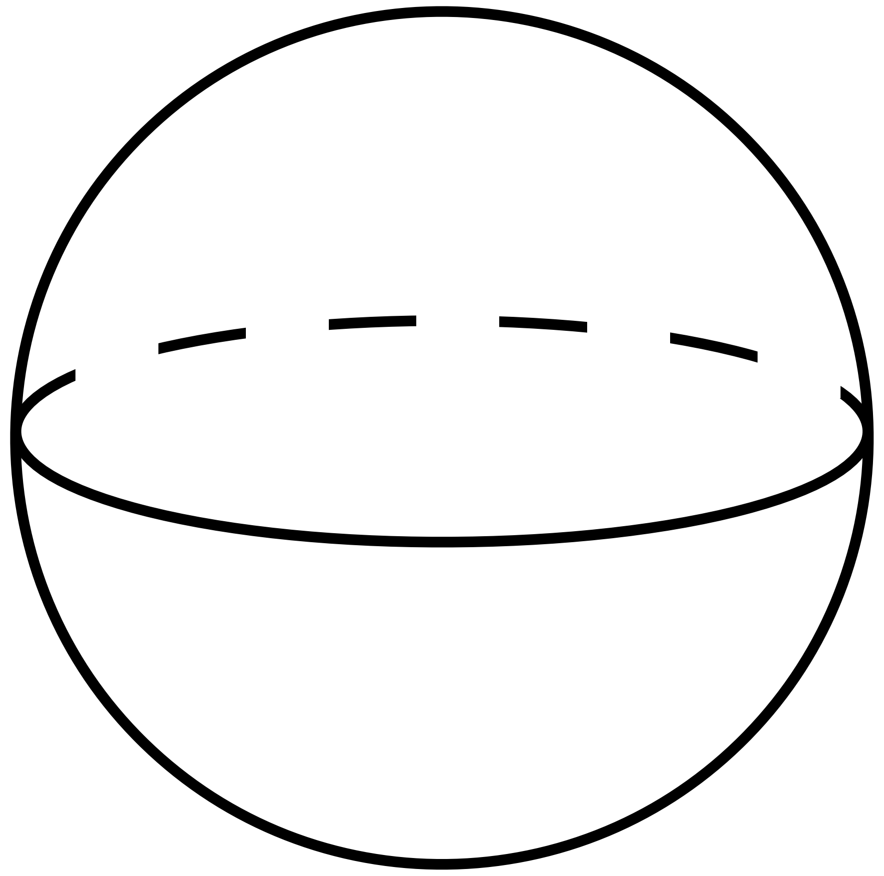


dS^4 Metamorphosis



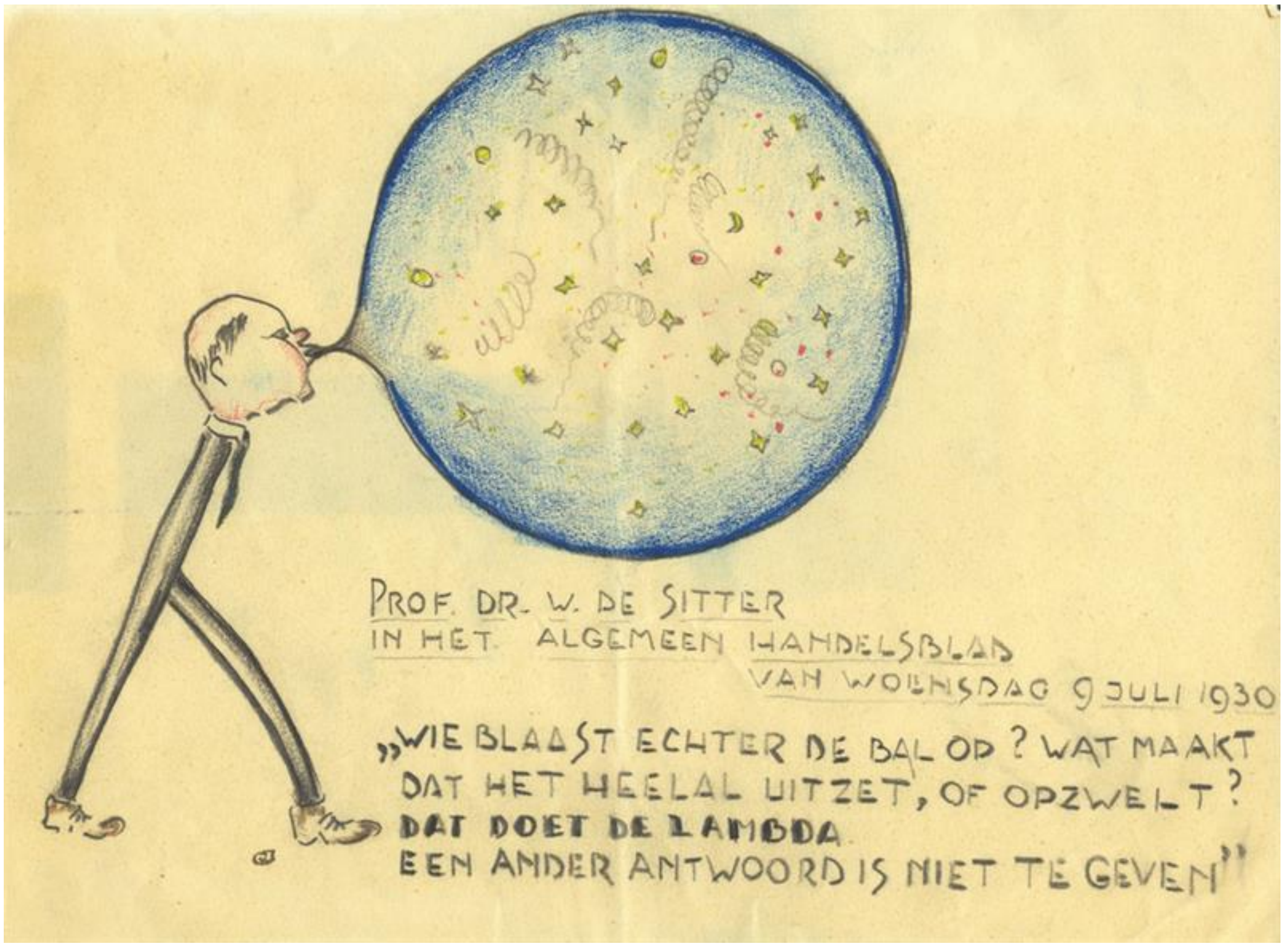
Beatrix Mühlmann
IAS | INSTITUTE FOR ADVANCED STUDY

Observers, wormholes and complex saddles in cosmology



ArXiv: 2602.19812 D. Anninos, C. Baracco, BM, V. Letsios

De Sitter quantum gravity



Models of de Sitter as tractable laboratories

dS_2 Timelike Liouville theory, Sine Dilaton gravity, JT gravity,...

[...]

dS_3 Complex Liouville string as matrix model/dS frame, timelike boundaries, DSSYK,...

[...]

dS_4 Late time wavefunction in dS_4 higher spin dual to Sp(N) CFT_3,...

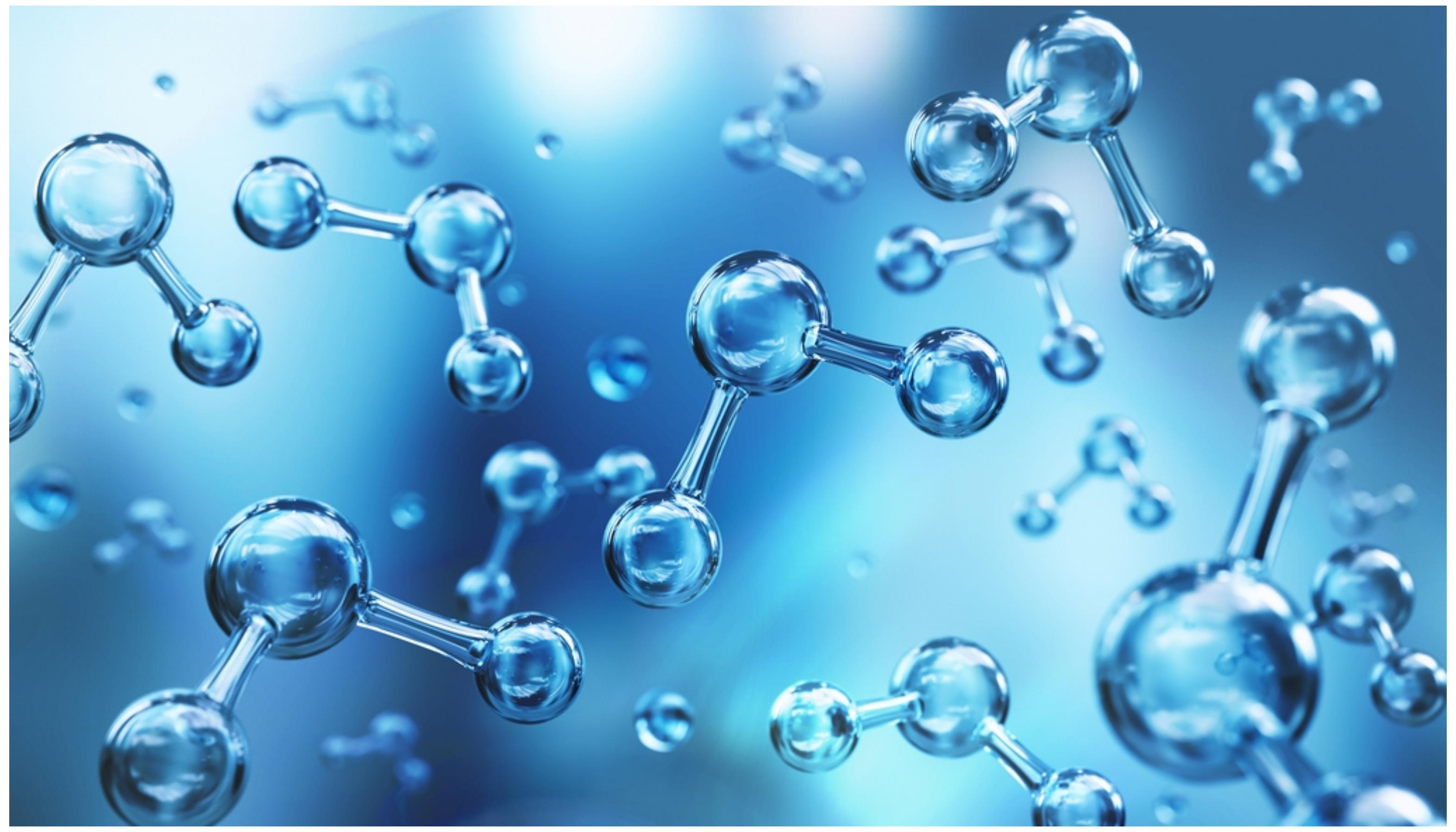
[...]

Sphere partition function

- Sphere partition function as gauge-invariant, systematically calculable “observable”
- Sphere is the Euclidean realisation of de Sitter space
- Conjecturally it encodes the de Sitter entropy

  [Gibbons-Hawking,…]

What are the molecules of our Universe?



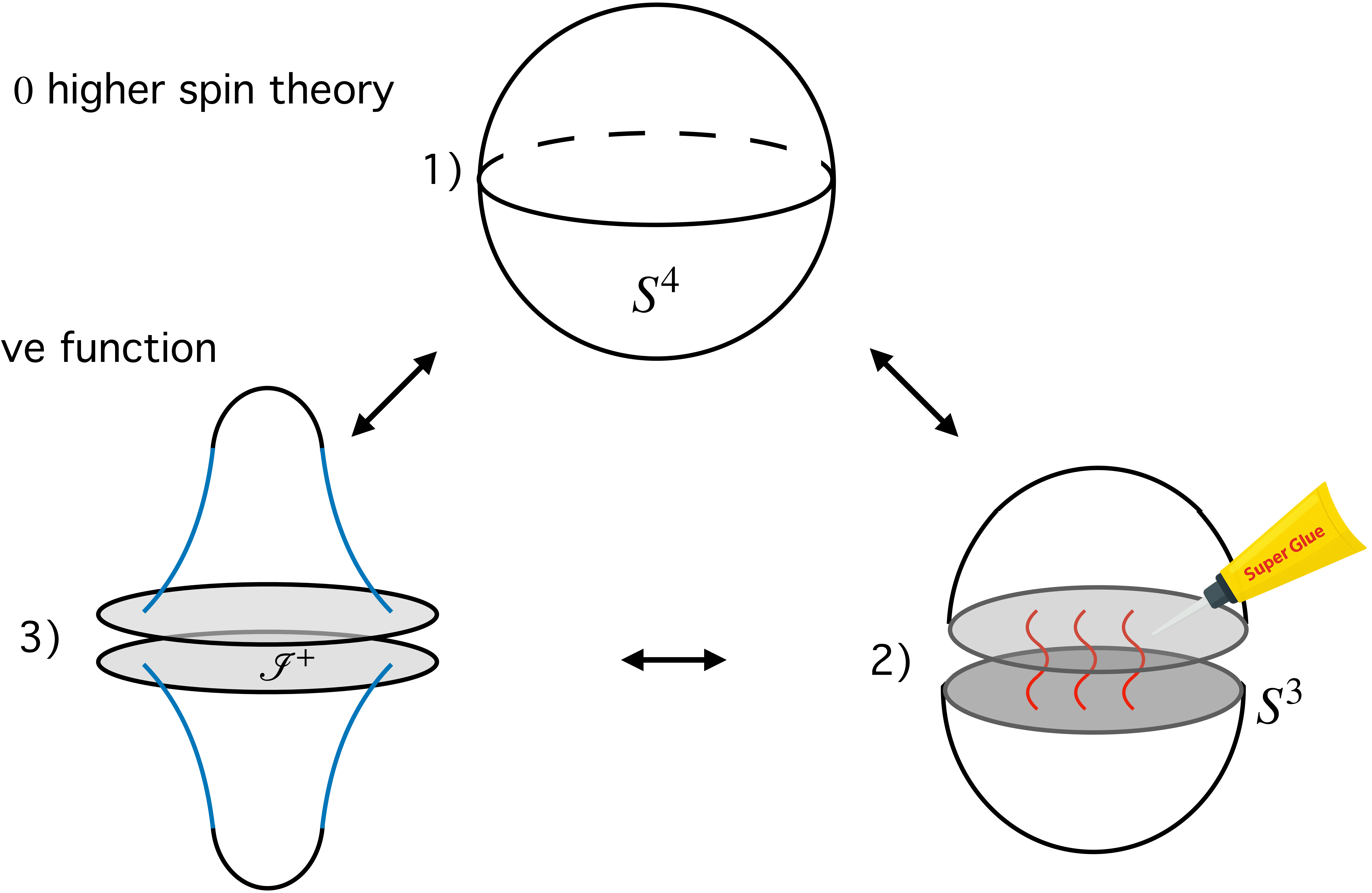
Outlook

0) Useful math framework

1) Sphere partition function of $\Lambda > 0$ higher spin theory

2) (Super)-gluing formula

3) dS/CFT and the norm of the wave function



Irreps

Classify free fields in dS according to the unitary irreducible representations of its isometry group

[Thomas, Newton, Dixmier, Harish-Chandra,……]

$dS_4 \Leftrightarrow SO(1,4)$

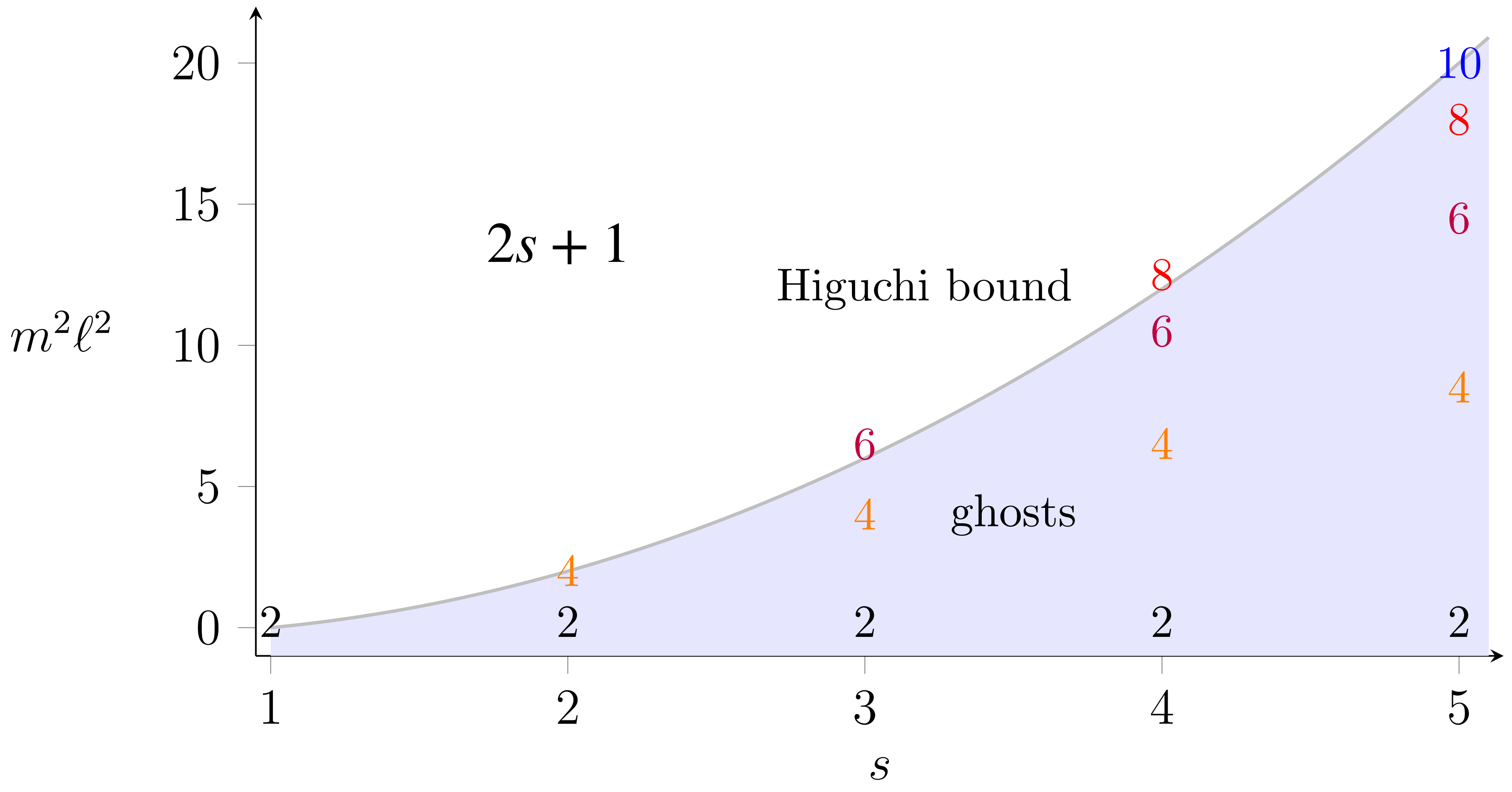
Irrep	Range of Δ	Range of s
π_ν	$\Delta = \frac{3}{2} + i\nu \,, \quad \nu \in \mathbb{R}_+$	$s = 0, 1, 2, \ldots$
γ_Δ	$0 < \Delta < 3$	$s = 0$
	$0 < \Delta < 2$	$s \geq 1$
$\mathscr{E}_{\Delta,0}$	$\Delta = 2 + p \,, \quad p \geq 1$	$s = 0$
$\mathscr{D}^{\pm}_{t,s}$	$\Delta = 2 + t \,, \quad t = 0, 1, \ldots, s-1$	$s \geq 1$

← Heavy fields

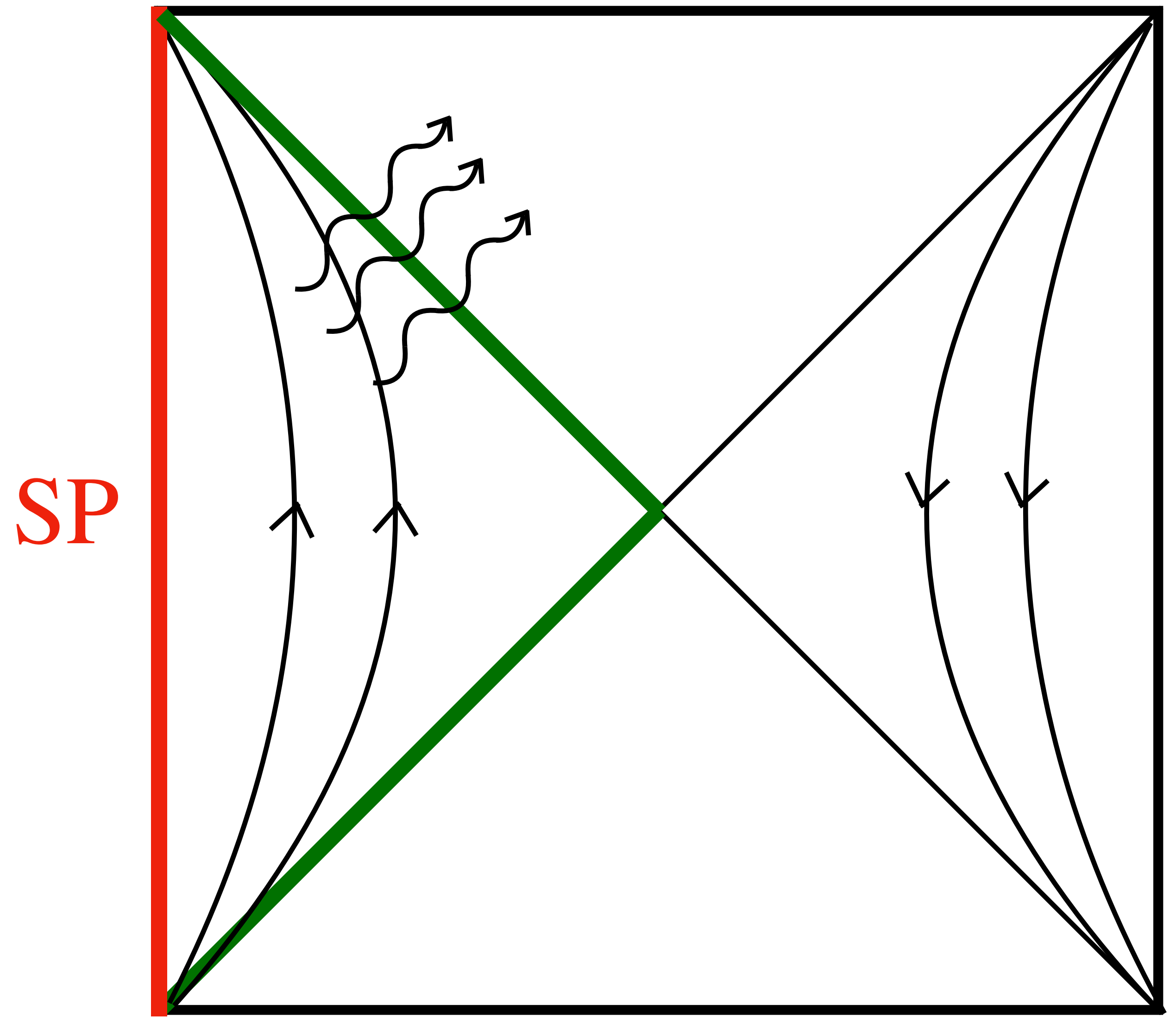
← Light fields

← Gauge fields

Partially massless fields



Characters



Harish-Chandra character for $\mathrm{SO}(1,d+1)$

[Harish-Chandra, Hirai,…]

$$\chi_{\Delta,s}(\mathfrak{t}) = \mathrm{tr}_{\Delta,s}\, e^{-i\mathfrak{t}H} = \int_{\mathbb{R}} d\omega\, \rho_{\Delta,s}(\omega)\, e^{-i\omega\mathfrak{t}} \,, \quad H \in \mathfrak{so}(1,1)$$

Example 1 Conformally coupled scalar in dS$_4$ $(m^2 = 2 \;\Leftrightarrow m^2 = \Delta(3-\Delta))$

$$\chi_{1,0}(t) = \frac{e^{-2\mathfrak{t}} + e^{-\mathfrak{t}}}{(1-e^{-\mathfrak{t}})^{\boxed{3}}} = e^{-\mathfrak{t}} + 4e^{-2\mathfrak{t}} + 9e^{-3\mathfrak{t}} + 16e^{-4\mathfrak{t}} + \ldots$$

Degeneracies of quasinormal modes

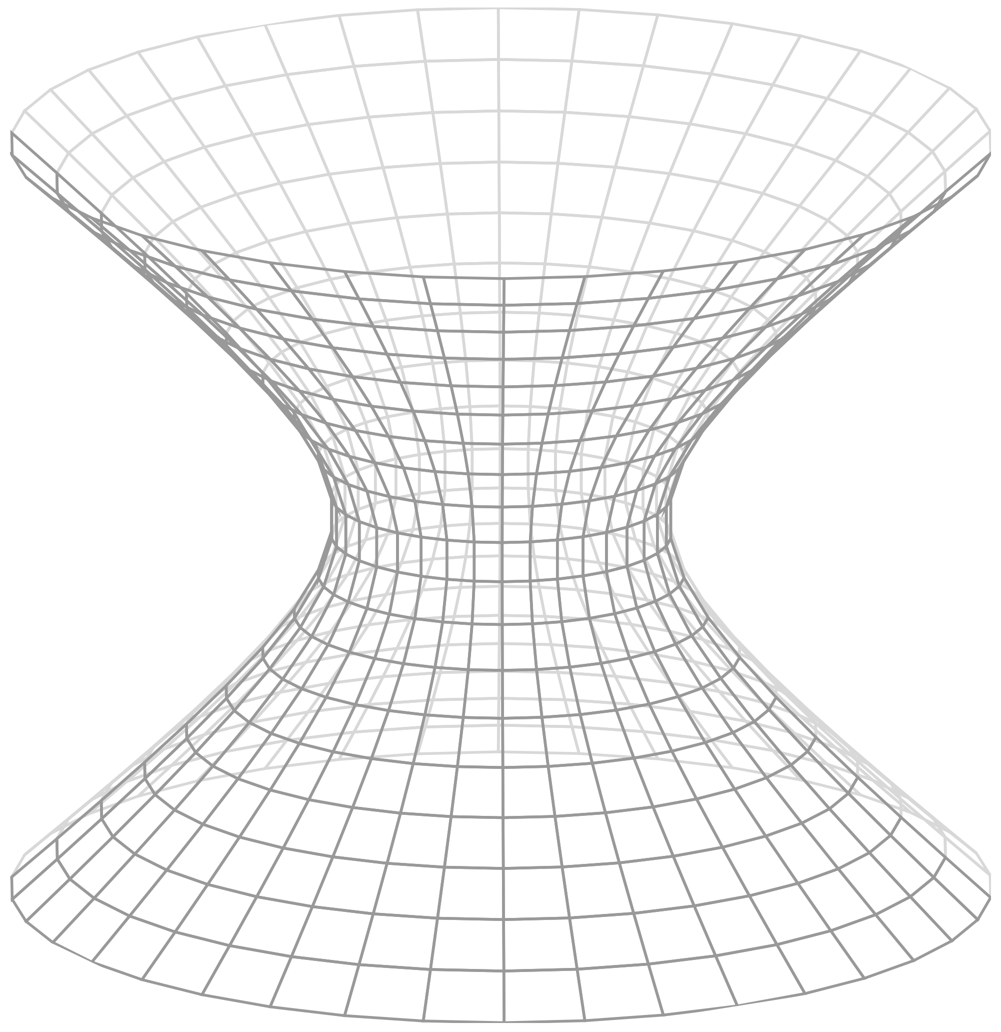
[Lopez-Ortega,…, Anninos-Denef-Law-Sun,…]

Example 2 Graviton in dS$_4$

$$\chi_{\mathrm{grav}}(\mathfrak{t}) = \chi_{\mathrm{bulk}}(\mathfrak{t}) - \chi_{\mathrm{edge}}(\mathfrak{t}) \qquad \chi_{\mathrm{bulk}}(\mathfrak{t}) = 2\frac{5e^{-3\mathfrak{t}} - 3e^{-4\mathfrak{t}}}{(1-e^{-\mathfrak{t}})^{\boxed{3}}} \,, \quad \chi_{\mathrm{edge}}(\mathfrak{t}) = \frac{10e^{-2\mathfrak{t}} - 2e^{-3\mathfrak{t}}}{(1-e^{-\mathfrak{t}})}$$

1-loop

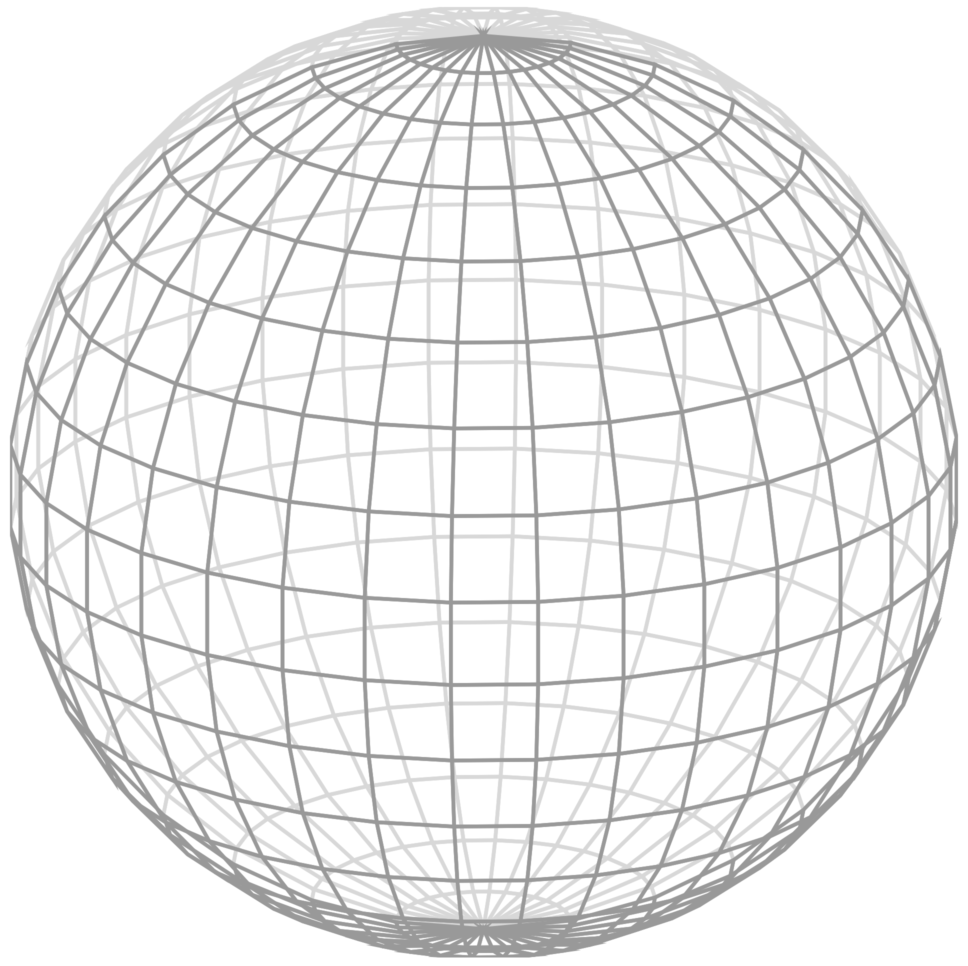
Lorentzian Harish-Chandra characters



⟷

[Anninos-Denef-Law-Sun,…]

Euclidean one-loop sphere partition functions



Example: $\log \mathcal{Z}^{(1)}_{\text{P.I.}} = \log \det^{-1/2}\left(-\nabla^2_{S^4}+2\right) = \int_0^\infty \frac{dt}{2t}\frac{1+e^{-t}}{1-e^{-t}}\chi_{1,0}(t)$

Minimal higher spin theory

Minimal higher spin theory consists of $\{\boxed{b}, \boxed{\underbrace{b_{\nu_1\nu_2}, b_{\nu_1\nu_2\nu_3\nu_4}}, \ldots}\}$

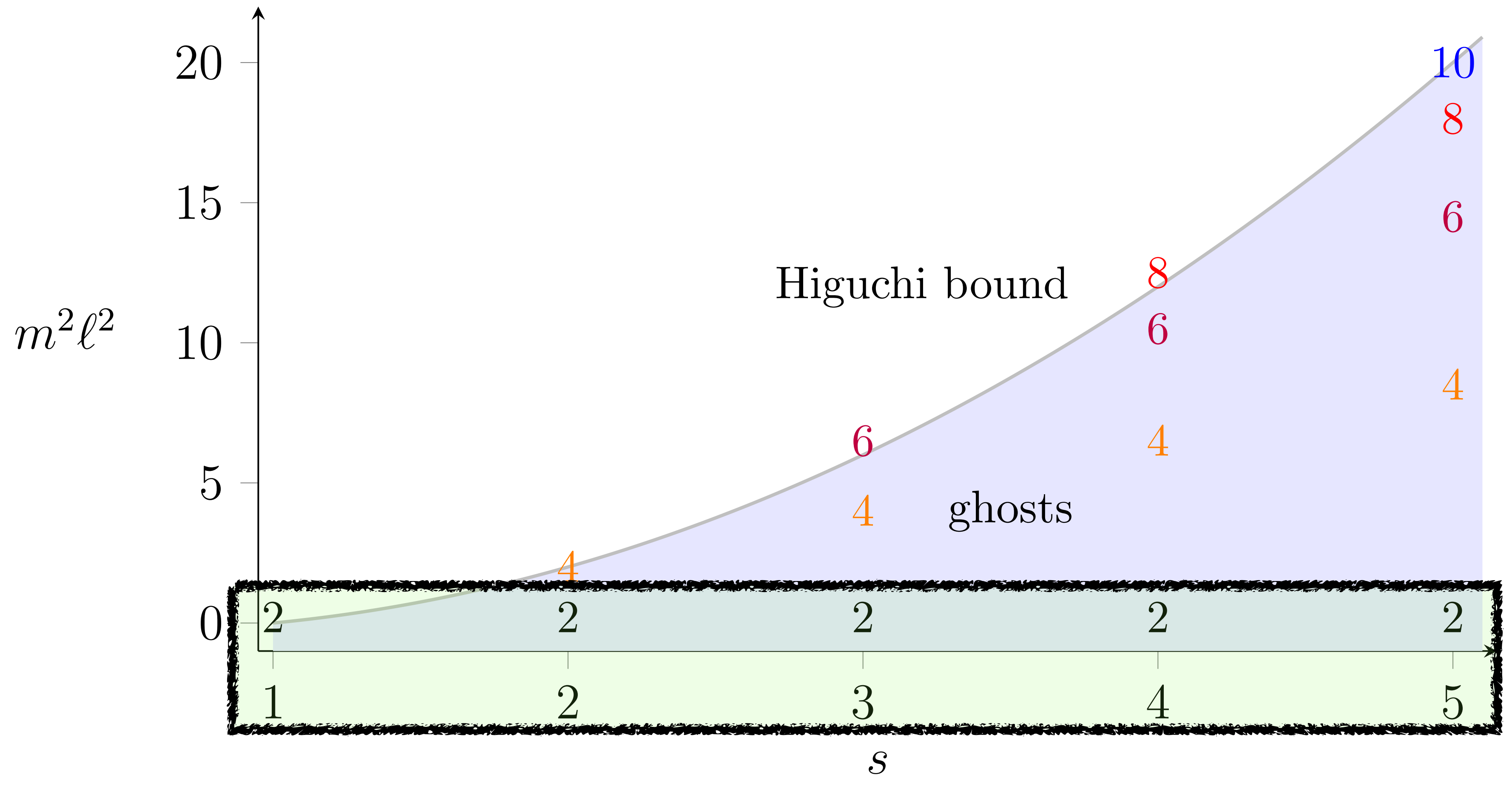
[Fronsdal, Fradkin-Vasiliev, Vasiliev,…]

Conformally coupled scalar

Highest depth partially massless fields

The Fronsdal fields $b_{\nu_1\ldots\nu_s}$

- totally symmetric & $b^{\nu\mu}{}_{\nu\mu\nu_5\ldots\nu_s} = 0$
- are invariant under $b_{\nu_1\ldots\nu_s} \to b_{\nu_1\ldots\nu_s} + \nabla_{(\mu}\zeta_{\nu_2\ldots\nu_s)}\ , \quad \zeta^{\mu}{}_{\mu\nu_3\ldots\nu_{s-3}} = 0$
- have two degrees of freedom
- are highest depth PMF with spin s and $\Delta = s+1$



Higher spin at 1-loop

What is the microscopic completion of $\boldsymbol{\Lambda > 0}$ higher spin on $\boldsymbol{S^4}$?

$$\log Z^{(1)}_{\text{hs}} = \int_0^\infty \frac{d\mathfrak{t}}{2\mathfrak{t}}\,\frac{1+\mathrm{e}^{-\mathfrak{t}}}{1-\mathrm{e}^{-\mathfrak{t}}} \sum_{s\in 2\mathbb{N}} \chi_{1+s,s}(\mathfrak{t}) = \int_0^\infty \frac{d\mathfrak{t}}{2\mathfrak{t}}\,\frac{1+\mathrm{e}^{-\mathfrak{t}}}{1-\mathrm{e}^{-\mathfrak{t}}}\left(\boxed{\frac{-\mathrm{e}^{-\mathfrak{t}}}{(1-\mathrm{e}^{-\mathfrak{t}})^{\boxed{2}}}} + 2\,\boxed{\frac{\mathrm{e}^{-\frac{3}{2}\mathfrak{t}}+\mathrm{e}^{-\frac{\mathfrak{t}}{2}}}{(1-\mathrm{e}^{-\mathfrak{t}})^{\boxed{2}}}}\right)$$

$$= \frac{\zeta(3)}{8\pi^2} + 2\left(\frac{3\zeta(3)}{16\pi^2} - \frac{1}{8}\log 2\right)$$

🟥 **The LHS organises itself into 3D characters**

🟨 Conformal higher spin theory

🟦 Conformally coupled scalar in dS_3

Sphere partition function

The sphere partition function also needs the on-shell action which famously is unknown

$$Z_{\mathrm{hs}}[S^4] \approx \mathrm{e}^{-\underset{\overset{\uparrow}{\boxed{?}}}{S^{(N)}_{\mathrm{EdS}}}} Z^{(1)}_{\mathrm{h.s.}}$$

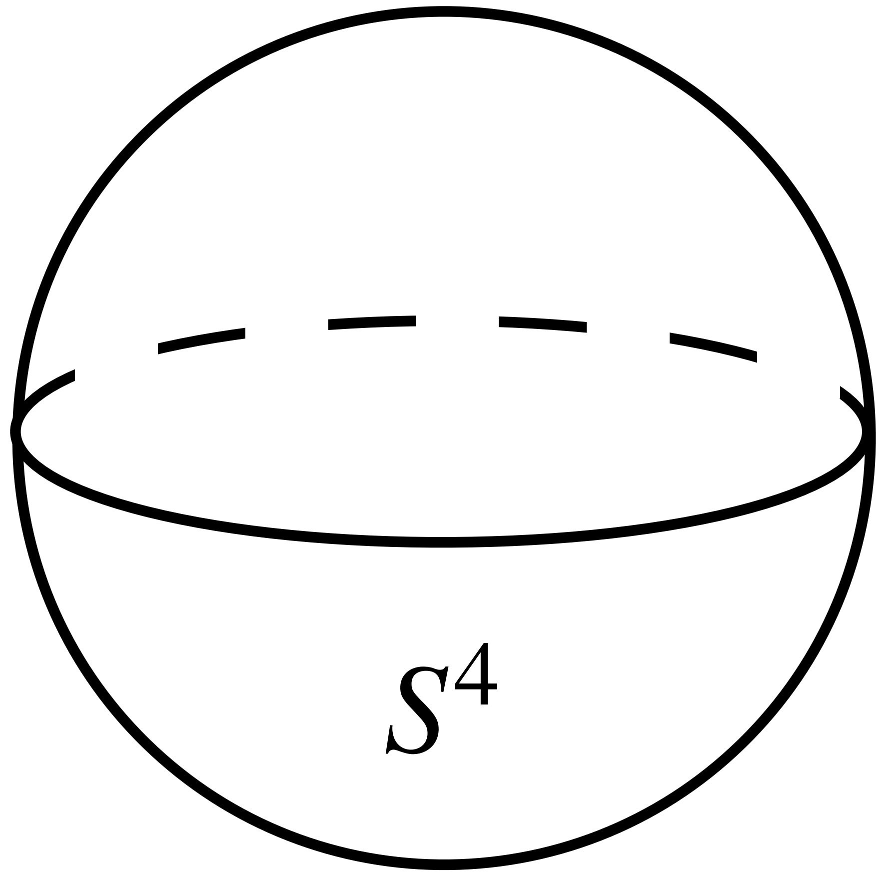
- ❗ AdS/CFT: Predicts $S^{(N)}_{\mathrm{EAdS}}$ with $N \approx (\Lambda G_N)^{-1}$
- ❗ Conjecture: $-S^{(N)}_{\mathrm{EdS}} = -2S^{(-N)}_{\mathrm{EAdS}} > 0$

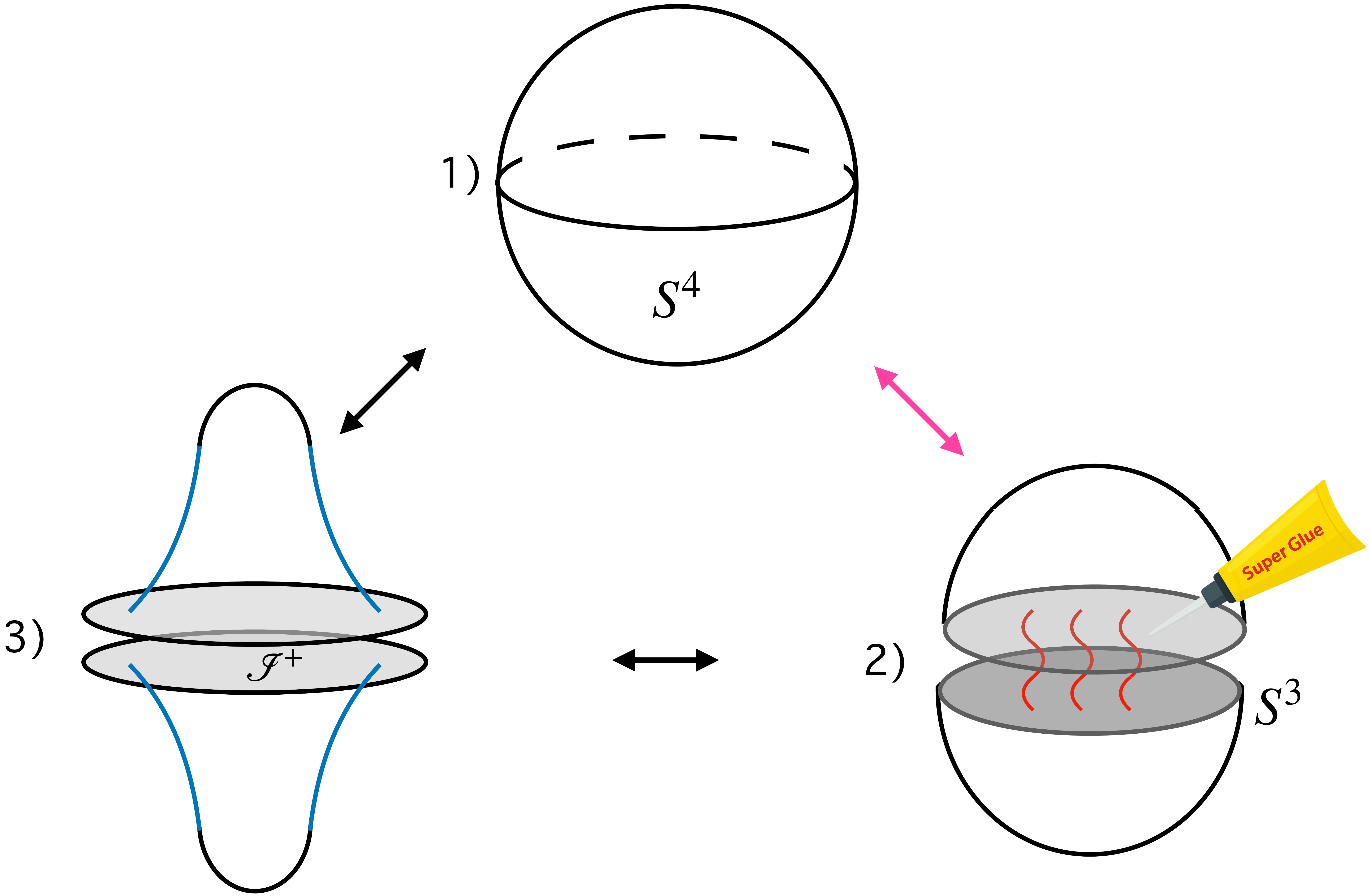
Together this leads to $$Z_{\mathrm{hs}}[S^4] \approx \frac{(-i)^{\mathscr{P}}}{\mathrm{vol}_N(G_{\mathrm{hs}})} \mathrm{e}^{2N\left(\frac{1}{8}\underbrace{\log 2 - \frac{3\zeta(3)}{16\pi^2}}_{>0}\right) + \frac{\zeta(3)}{8\pi^2}}$$

$\underline{\mathscr{P}}$ The phase cancels in some cases

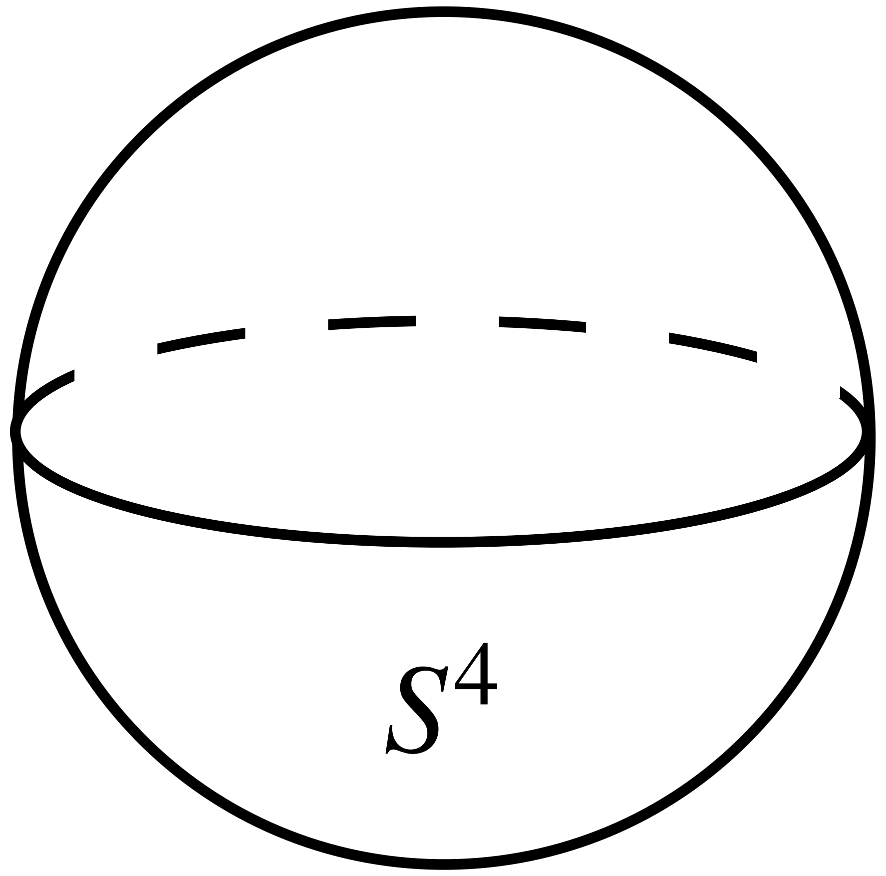
[Giombi-Sun, Anninos-Baracco-BM-Letsios,…]

$\underline{G_{\mathrm{hs}}}$ Generated by spin s Killing tensors

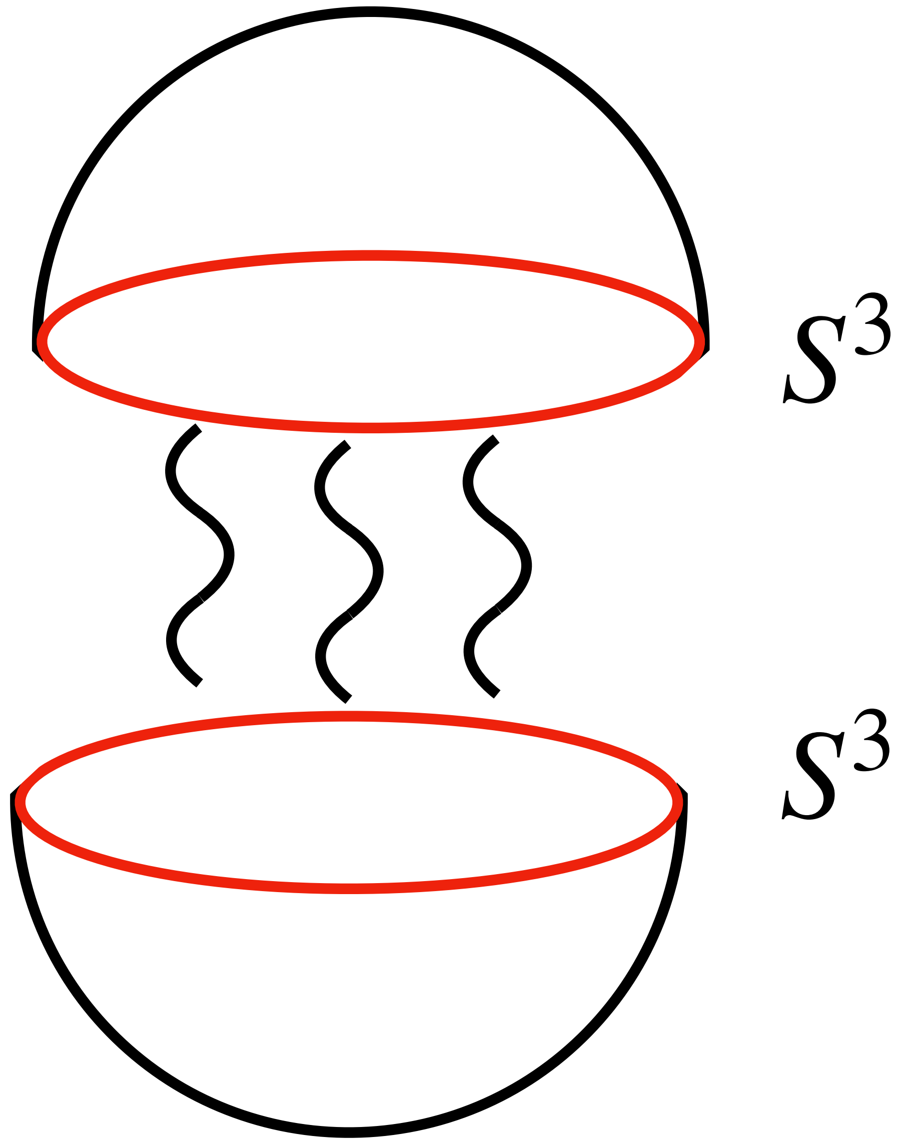



1)
S^4
2)
Super Glue
S^3
3)
$\mathscr{I}^+$

Conjecture



$$= \quad Z_{\rm hs}[S^4] = \frac{(-i)^{\mathscr{P}}}{\mathrm{vol}_N(G_{\rm hs})}\int [\mathscr{D}B]\left|Z_{Sp(N)}[B]\right|^2 \quad =$$



$\underline{Z_{Sp(N)}}$ Partition function of $Sp(N)$ vector model build from *anticommuting* scalars

$$Z_{Sp(N)}[B] = \int [\mathscr{D}\chi_I]\mathrm{e}^{-\int_{S^3}\chi^I(-\nabla_c^2)\chi^J\Omega_{IJ}+b\mathscr{O}+b^{ij}\mathscr{O}_{ij}+\dots}$$

$$\mathscr{O} = \chi^I\chi^J\Omega_{IJ} \qquad I = 1,\dots,N$$

$$\mathscr{O}_{ij} = \chi^I\partial_i\partial_j\chi^J\Omega_{IJ}$$

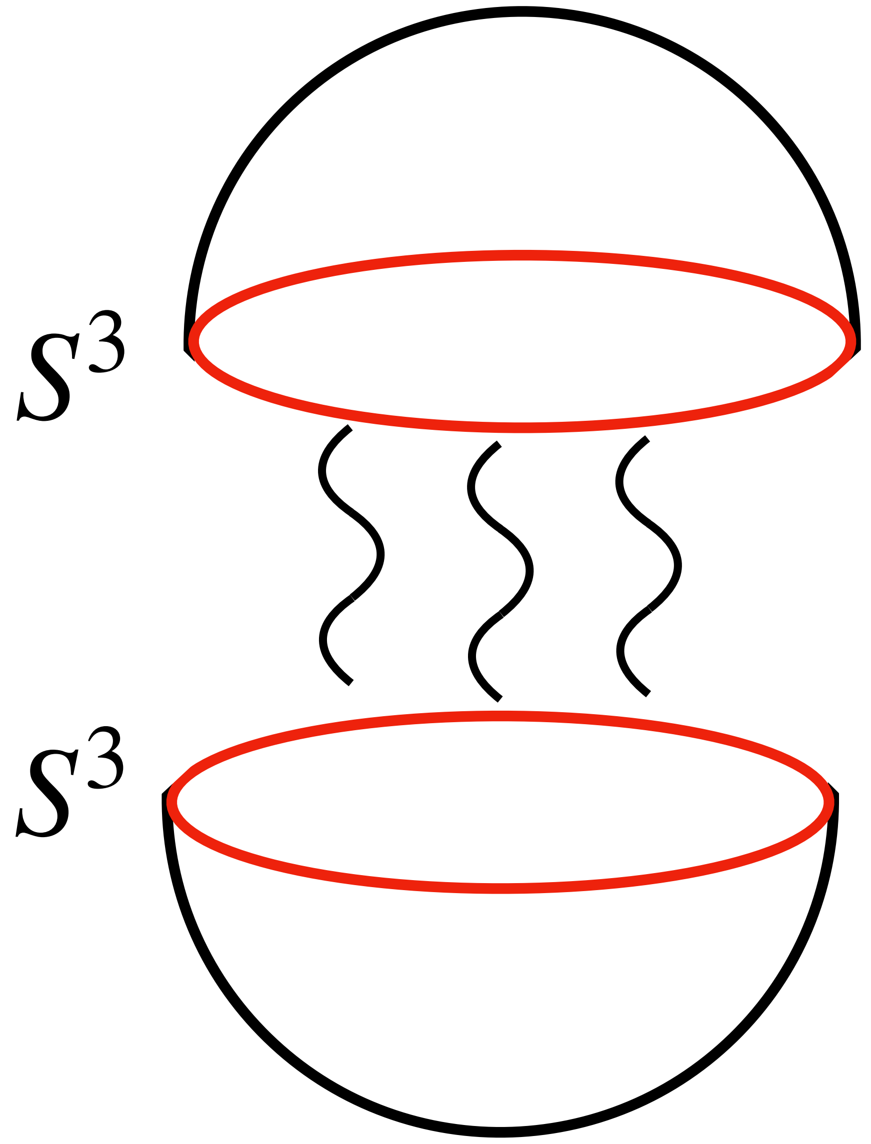
$$\vdots$$

$\underline{B}$ bilocal field which sources the $Sp(N)$ invariant operators

$\underline{G_{\rm hs}}$ 4D Vasiliev algebra = 3D Eastwood algebra

(SO(1,4) = dS$_4$ isometry group and 3D Euclidean conformal group)

Gluing formula

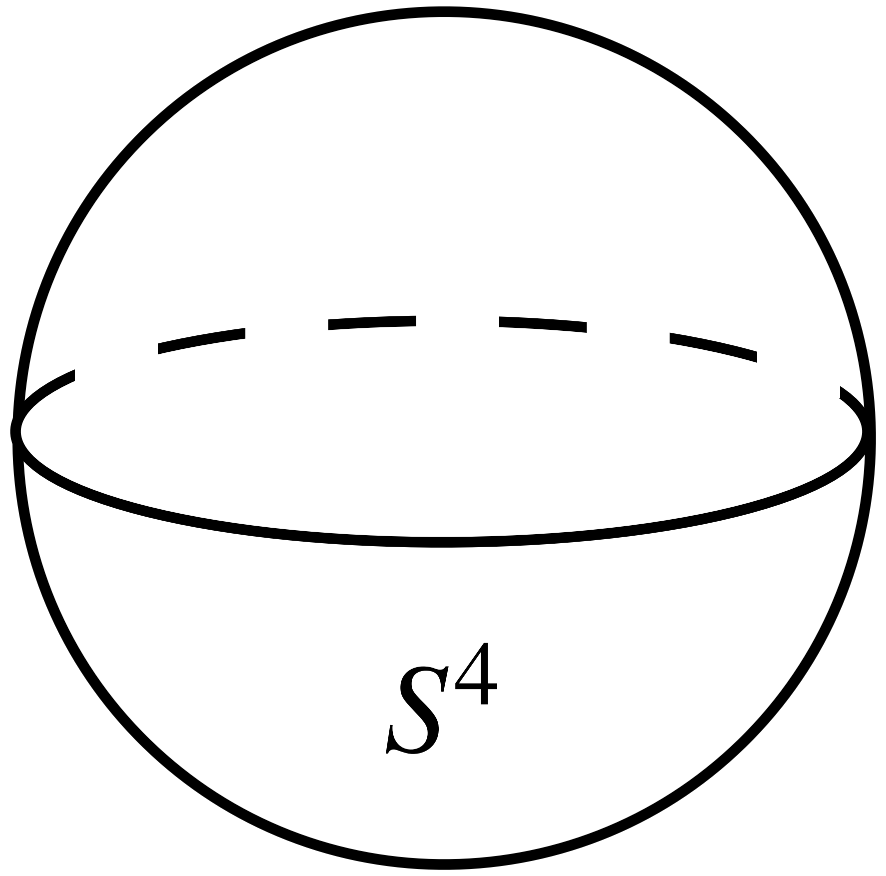


$$= \frac{(-i)^{\mathscr{P}}}{\mathrm{vol}_N(G_{\mathrm{hs}})}\int [\mathscr{D}B]\left|Z_{Sp(N)}[B]\right|^2 = \frac{(-i)^{\mathscr{P}}}{\mathrm{vol}_N(G_{\mathrm{hs}})}\int [\mathscr{D}B]\det{}^N\left(-\nabla_c^2+B\right)$$

$$\approx \frac{(-i)^{\mathscr{P}}}{\mathrm{vol}_N(G_{\mathrm{hs}})} \times \boxed{\det{}^N(-\nabla_c^2)} \times \boxed{\begin{array}{c}\text{higher spin two-point}\\ \text{function}\end{array}}$$

[Giombi-Klebanov-Pufu-Safdi-Tarnopolsky,…]

$$= \frac{(-i)^{\mathscr{P}}}{\mathrm{vol}_N(G_{\mathrm{hs}})} e^{2N(\frac{1}{8}\log 2-\frac{3\zeta(3)}{16\pi^2})+\frac{\zeta(3)}{8\pi^2}} =$$



Extension

Integer + half-integer higher spin fields

$$\{\boxed{b},\boxed{b_{\nu_1}, \underbrace{b_{\nu_1\nu_2}}, b_{\nu_1\nu_2\nu_3}, \dots}\} \qquad + \qquad \{\boxed{\psi_\alpha, \underbrace{\psi_{\alpha\mu_1}}, \psi_{\alpha\mu_1\mu_2}, \dots}\}$$

Conformally coupled scalar

Highest depth PMF $s = 1,2,\dots$

Massless Dirac

Complex massless gauge fields $s = \frac{3}{2}, \frac{5}{2}, \dots$

Irrep	Range of Δ	Range of s
π_ν	$\Delta = \frac{3}{2} + i\nu \ , \quad \nu \in \mathbb{R}_+$	$s = 0, 1, 2, \dots$
γ_Δ	$0 < \Delta < 3$	$s = 0$
	$0 < \Delta < 2$	$s \geq 1$
$\mathcal{D}^{\pm}_{t,s}$	$\Delta = 2 + t \ , \quad t = 0, 1, \dots, s-1$	$s \geq 1$

Irrep	Range of Δ	Range of s
π_ν	$\Delta = \frac{3}{2} + i\nu \ , \quad \nu \in \mathbb{R}_+$	$s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$
	✗	
$\mathcal{D}^{\pm}_{t,s}$	$\Delta = 2 + t \ , \quad t = -\frac{1}{2}, \frac{1}{2}, \dots, s-1$	$s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$

Superglubits

Combining the on-shell action and the 1-loop contribution leads to

$$Z^{(N)}_{\mathcal{N}=2\text{ hs}}[S^4] \approx \boxed{\frac{1}{\mathrm{vol}_N(G_{\mathrm{s.hs}})}} \times \boxed{\mathrm{e}^{-S^{(N)}_{\mathrm{EdS}}}} \times \boxed{Z^{(1)}_{\mathcal{N}=2\text{ hs}}} = 2^N \times \frac{N^{-\frac{1}{16}}}{\mathrm{vol}(G_{\mathrm{s.hs}})}$$

🟦 $\mathrm{vol}_N(G_{\mathrm{s.hs}}) = N^{\frac{1}{16}}\mathrm{vol}(G_{\mathrm{s.hs}})$

$\uparrow$

Σ Integer+half-integer KT/2

🟨 $-S^{(N)}_{\mathrm{EdS}} = N\log(2) - \frac{\zeta(3)}{4\pi^2}$

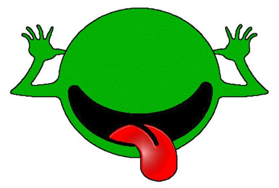
🟪 $\log Z^{(1)} = \frac{\zeta(3)}{4\pi^2}$

Ratio is independent of $G_{\mathrm{s.hs}}$

$$r_{N,M} \equiv \frac{Z^{(N)}_{\mathcal{N}=2\text{ hs}}[S^4]}{Z^{(M)}_{\mathcal{N}=2\text{ hs}}[S^4]}$$

$N = (432)\ , \quad M = (432)2^{16}$

$\Rightarrow \log(r_{N,M}) = 10^{42}$

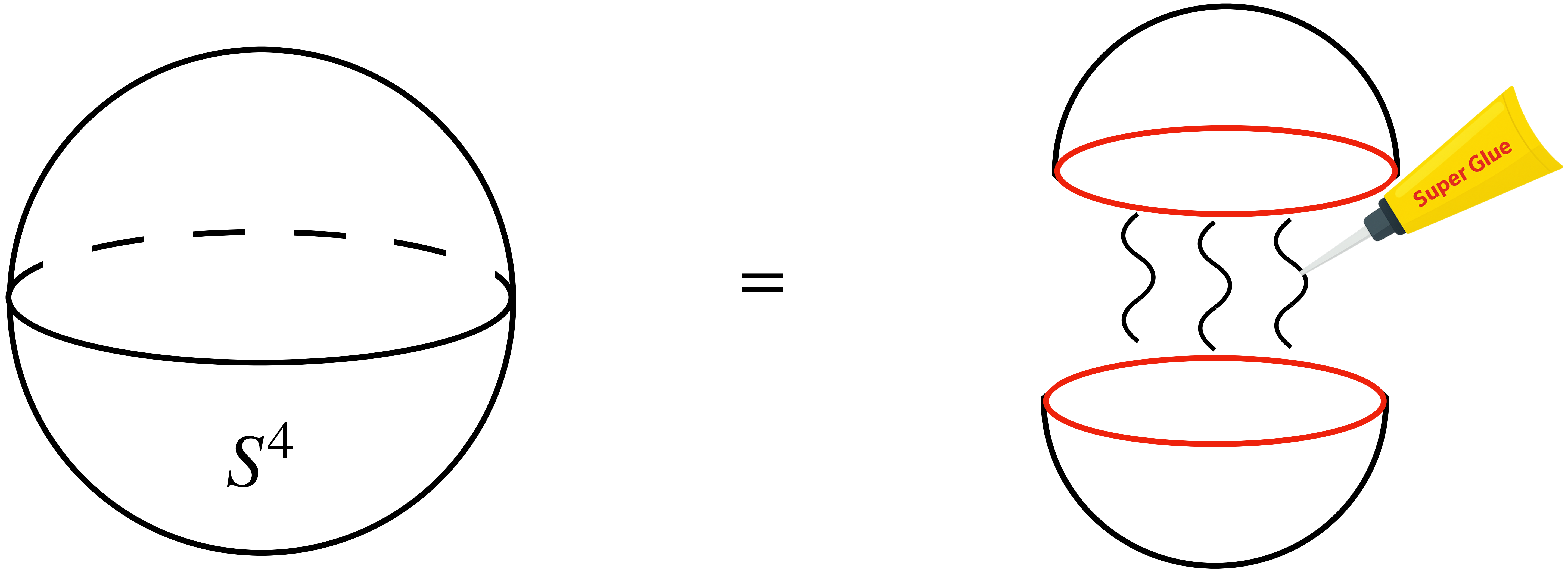


Supergluing formula

$$Z_{\mathcal{N}=2\ \mathrm{hs}}[S^4] = \frac{1}{\mathrm{vol}_N(G_{\mathrm{s.hs}})} \int [\mathscr{D}B_s] \left| Z_{U(N)}[B_s] \right|^2$$

$\underline{Z_{U(N)}}$ Partition function of N free *anticommuting* complex scalars & N free *commuting* Dirac spinors

$$Z_{U(N)}[B_s] = \int [\mathscr{D}\underset{\substack{\uparrow \\ \text{chiral superfield}}}{\Phi_I}][\mathscr{D}\underset{\substack{\uparrow \\ \text{anti-chiral superfield}}}{\widetilde{\Phi}_I}]\mathrm{e}^{-\int \mathrm{d}z \widetilde{\Phi}_I(\tilde{z})\Phi_I(z) + \int \mathrm{d}z_1 \mathrm{d}z_2 \widetilde{\Phi}_I(\tilde{z}_1) \underset{\substack{\uparrow \\ \text{bi-local superfield}}}{B_s}(z_1;\tilde{z}_2)\Phi_I(z_2)}$$



Outlook

- Gluing formula for general dimension? [WIP Anninos-Baracco-BM-Letsios…]
- $2^N = \lim_{\beta \to 0} (e^{\beta\omega/2} + e^{-\beta\omega/2})^N \quad \Rightarrow$ microscopic picture?
- Beyond one-loop
- Timelike Toda as a $2D$ toy model [WIP BM…]
  - $\mathcal{N} = 2$ Liouville [WIP BM- Zikopolous…]

⋮

Relation to dS/CFT

Higher spin in Lorentzian dS_4

A different perspective:

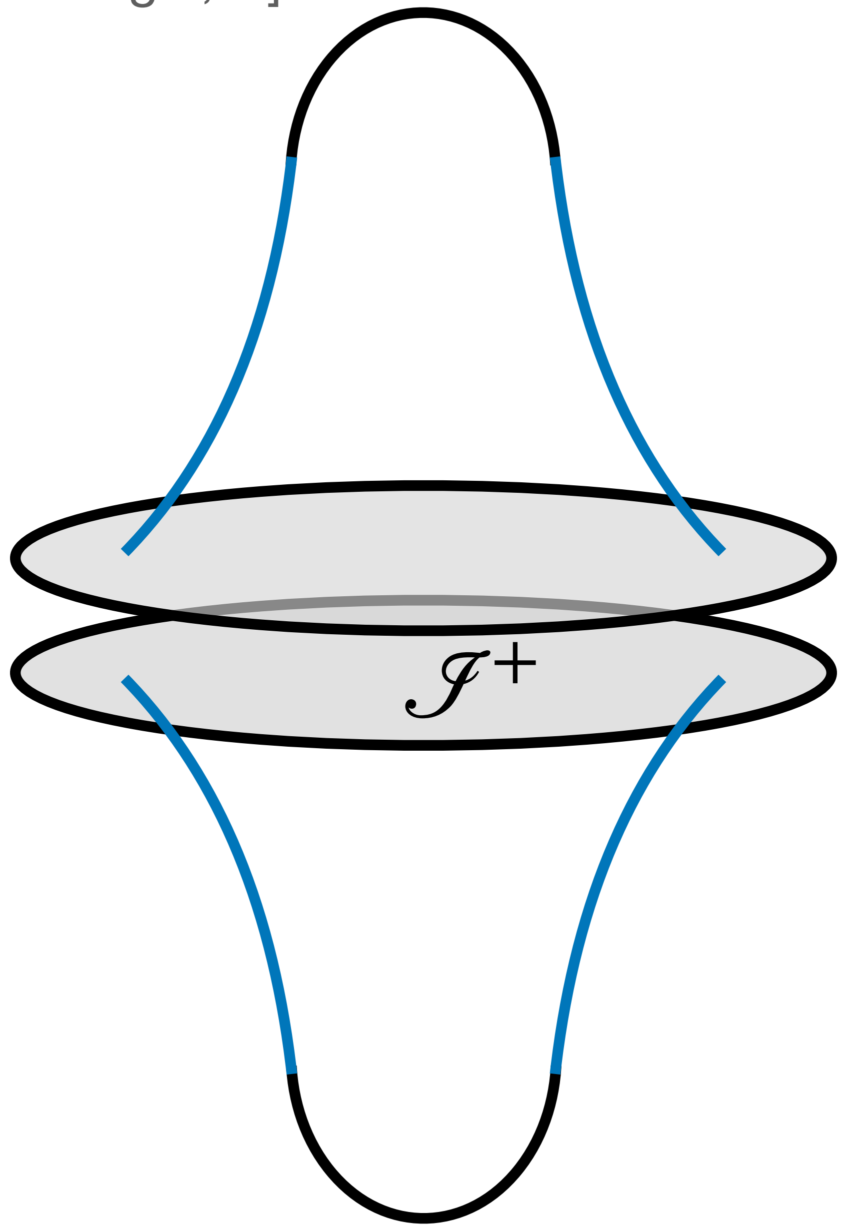
dS/CFT: HH wave function $\longleftrightarrow$ CFT partition function at $\mathscr{I}^+$

[Maldacena,…]

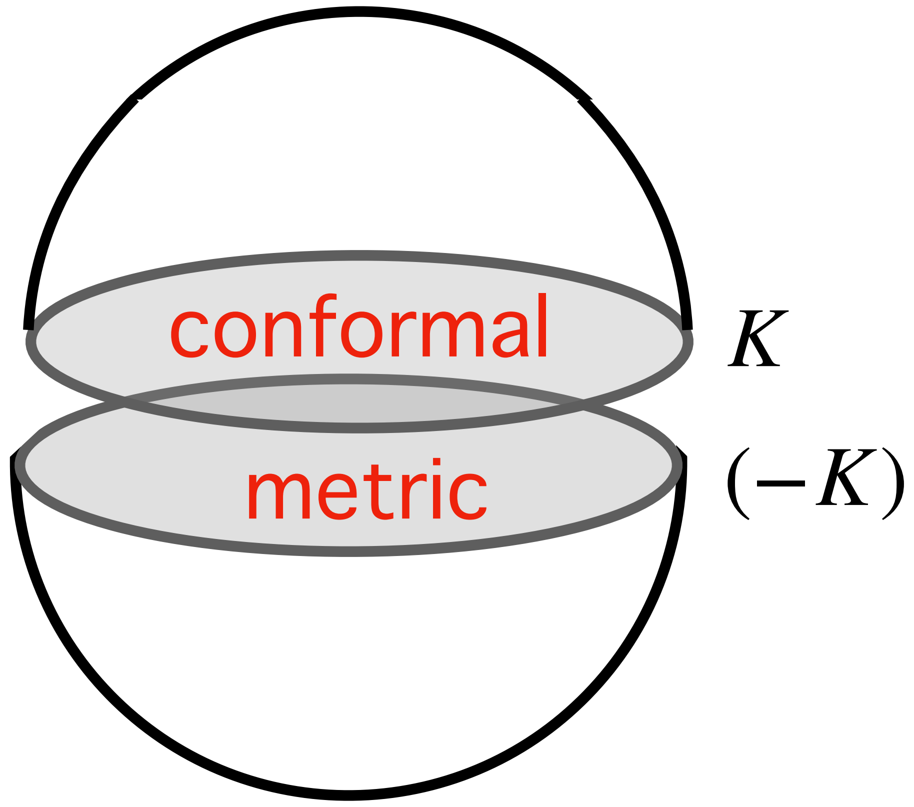
dS isometries become conformal transformations at $\mathscr{I}^+$

Higher spin dS/CFT: $\Psi_{\rm HH}[B_{\boldsymbol{\mu}}] = Z_{Sp(N)}[B_{\boldsymbol{i}}]$

[Anninos-Hartman-Strominger,…]



$$\langle \Psi_{\rm HH} | \Psi_{\rm HH} \rangle \overset{?}{=} Z_{\rm hs}[S^4]$$



Norm at finite boundary with York time K as time parameter

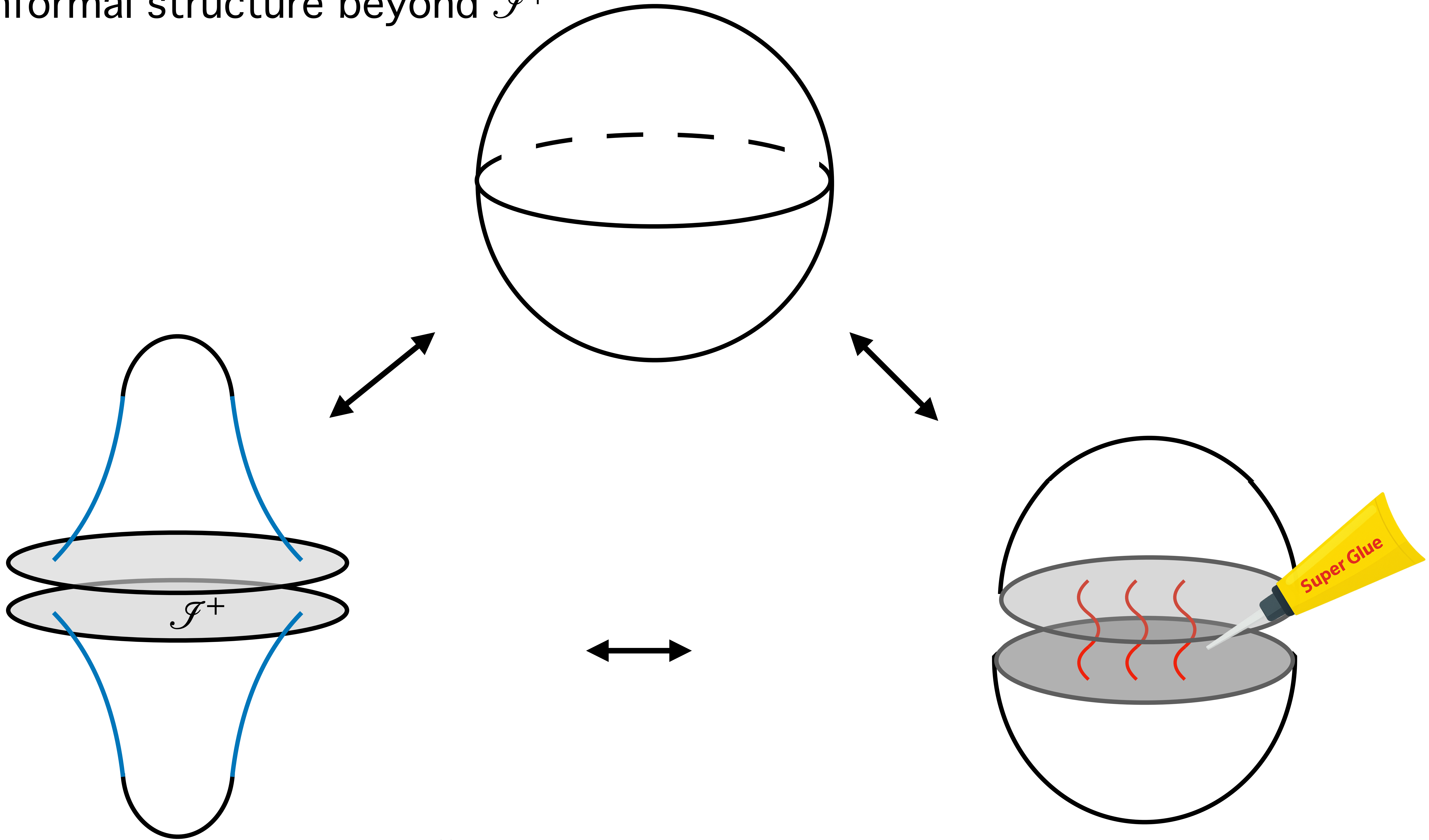
[WIP Blommaert-BM-Verlinde…]

Summary

1) We developed a machinery to access the $\Lambda > 0$ higher spin sphere partition function

2) Gluing formula reveals conformal structure beyond $\mathscr{I}^+$

3) Hints of a norm?



Thank you!