IAS | INSTITUTE FOR ADVANCED STUDY

The observer's no-boundary state and axion wormholes

Erez Y. Urbach

[2505.14771] Blommaert, Kudler-Flam, EYU

[26??.?????] Blommaert, Kudler-Flam, Narovlansky, EYU

Observers, wormholes and complex saddles in cosmology, EPFL

Quantum gravity in closed universes

- In AdS/CFT the holographic dual is quantum mechanical. This allowed us to learn a lot about the non-perturbative aspects of quantum gravity. Concretely, we can unambiguously define bulk operators by dressing them to the fixed conformal boundary.

Quantum gravity in closed universes

- In AdS/CFT the holographic dual is quantum mechanical. This allowed us to learn a lot about the non-perturbative aspects of quantum gravity. Concretely, we can unambiguously define bulk operators by dressing them to the fixed conformal boundary.
- For closed universes (such as de Sitter), we lack both a holographic description and a fixed boundary. It has been known for many years that in order to meaningfully define observables one needs to **introduce an ideal clock/observer** [Page-Wooters,...].

Quantum gravity in closed universes

- In AdS/CFT the holographic dual is quantum mechanical. This allowed us to learn a lot about the non-perturbative aspects of quantum gravity. Concretely, we can unambiguously define bulk operators by dressing them to the fixed conformal boundary.
- For closed universes (such as de Sitter), we lack both a holographic description and a fixed boundary. It has been known for many years that in order to meaningfully define observables one needs to **introduce an ideal clock/observer** [Page-Wooters,...].
- [Chandrasekaran, Longo, Penington & Witten, '22] Studied an ideal clock in de Sitter space at $G = 0$. This allowed them to define an (diff-invariant) **algebra of observables for the observer's static patch**.

Quantum gravity in closed universes

- In AdS/CFT the holographic dual is quantum mechanical. This allowed us to learn a lot about the non-perturbative aspects of quantum gravity. Concretely, we can unambiguously define bulk operators by dressing them to the fixed conformal boundary.
- For closed universes (such as de Sitter), we lack both a holographic description and a fixed boundary. It has been known for many years that in order to meaningfully define observables one needs to **introduce an ideal clock/observer** [Page-Wooters,...].
- [Chandrasekaran, Longo, Penington & Witten, '22] Studied an ideal clock in de Sitter space at $G = 0$. This allowed them to define an (diff-invariant) **algebra of observables for the observer's static patch**.
- The algebra was found to be type II_1, which admits a "renormalized-entropy" function that agrees with the area law:

$$S_{\mathrm{vN}}(\rho) = \underbrace{\frac{\delta A_{\mathrm{dS}}}{4G} + \delta S_{\mathrm{QFT}}}_{O(1)} + C. \tag{1}$$

Quantum gravity in closed universes

- In this algebra, the Hartle-Hawking/no-boundary state (Bunch-Davis) is identified with the identity operator:

$$\rho_{\text{NB}} = \mathbb{1}. \tag{2}$$

  In other words, **the no-boundary state is the maximum entropy state** for the observer's algebra.

Quantum gravity in closed universes

- In this algebra, the Hartle-Hawking/no-boundary state (Bunch-Davis) is identified with the identity operator:

$$\rho_{\text{NB}} = \mathbb{1}. \tag{2}$$

  In other words, **the no-boundary state is the maximum entropy state** for the observer's algebra.

- **Intuition:** from the observer's perspective, the state thermalizes at late times to the Bunch-Davis state. By the second law, it is the **maximum entropy state**.

Quantum gravity in closed universes

- In this algebra, the Hartle-Hawking/no-boundary state (Bunch-Davis) is identified with the identity operator:

$$\rho_{\text{NB}} = \mathbb{1}. \tag{2}$$

  In other words, **the no-boundary state is the maximum entropy state** for the observer's algebra.

- **Intuition:** from the observer's perspective, the state thermalizes at late times to the Bunch-Davis state. By the second law, it is the **maximum entropy state**.
- The construction is perturbative $G \ll 1$, and allows us to only compute entropy differences from the Bunch-Davis state.

Quantum gravity in closed universes

- In this algebra, the Hartle-Hawking/no-boundary state (Bunch-Davis) is identified with the identity operator:

$$\rho_{\text{NB}} = \mathbb{1}. \tag{2}$$

  In other words, **the no-boundary state is the maximum entropy state** for the observer's algebra.

- **Intuition:** from the observer's perspective, the state thermalizes at late times to the Bunch-Davis state. By the second law, it is the **maximum entropy state**.
- The construction is perturbative $G \ll 1$, and allows us to only compute entropy differences from the Bunch-Davis state.
- As a result, we can't fix the additive constant in S_{vN}. More generally, unable to compare entropies around different backgrounds.

Quantum gravity in closed universes

- In this algebra, the Hartle-Hawking/no-boundary state (Bunch-Davis) is identified with the identity operator:

$$\rho_{\text{NB}} = \mathbb{1}. \tag{2}$$

  In other words, **the no-boundary state is the maximum entropy state** for the observer's algebra.

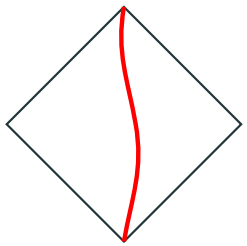
- **Intuition:** from the observer's perspective, the state thermalizes at late times to the Bunch-Davis state. By the second law, it is the **maximum entropy state**.
- The construction is perturbative $G \ll 1$, and allows us to only compute entropy differences from the Bunch-Davis state.
- As a result, we can't fix the additive constant in S_{vN}. More generally, unable to compare entropies around different backgrounds.
- We would like a "less-perturbative" principle by which we can define traces and entropies.

A background-independent algebra for observers

- [Witten ’23] considered any background together with an ideal clock.

A background-independent algebra for observers

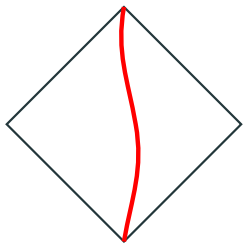
- **[Witten ’23] considered any background together with an ideal clock**.
- One can unambiguously define an algebra for the observer’s causal region.



This algebra captures all the possible experiments one can do along the clock’s worldline.

A background-independent algebra for observers

- **[Witten '23] considered any background together with an ideal clock**.
- One can unambiguously define an algebra for the observer's causal region.

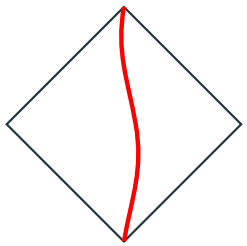


This algebra captures all the possible experiments one can do along the clock's worldline.

- **Conjecture:** ρ_{NB} is a universal maximum entropy state.

A background-independent algebra for observers

- **[Witten '23] considered any background together with an ideal clock**.
- One can unambiguously define an algebra for the observer's causal region.



This algebra captures all the possible experiments one can do along the clock's worldline.

- **Conjecture:** ρ_{NB} is a universal maximum entropy state.
- This would allow us to unambiguously define entropies as

$$S(\rho) = -\langle \rho \log \rho \rangle_{\text{NB}} \equiv -S_{\text{rel}}(\rho \,\|\, \rho_{\text{NB}}). \tag{3}$$

Fixing the entropy's additive constant, with $C = A_{\text{dS}}/(4G)$ for dS (CLPW).

Plan

1. Give a path integral argument for Witten's conjecture for **closed universes with full causal access**.
   Suggest that a Lorentzian prescription is the correct path integral definition for ρ_{NB}.

Plan

1. Give a path integral argument for Witten's conjecture for **closed universes with full causal access**.
   Suggest that a Lorentzian prescription is the correct path integral definition for ρ_{NB}.
2. Test the prescription for the case of the **axion wormholes**.
   The Lorentzian contour will avoid a puzzle with the naive Euclidean prescription, and reproduce the expected answer.

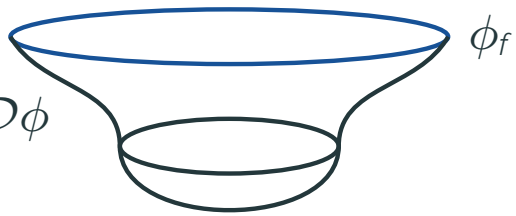
The observer’s no-boundary state

The no-boundary proposal

[Hartle, Hawking, Hertog, ...]

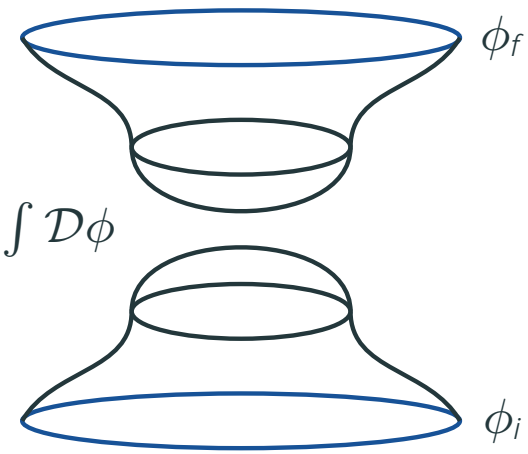
- For a given configuration ϕ_f, the no-boundary state $\Psi[\phi_f]$ is given by all the compact geometries that end on ϕ_f:

$$\Psi[\phi_f] = \int \mathcal{D}\phi$$



- Similarly, we can write the (pure) density matrix $\rho_{\text{NB}}(\phi_f, \phi_i)$ as a path integral with two boundaries:

$$\rho_{\text{NB}}(\phi_f, \phi_i) = \int \mathcal{D}\phi$$



The observer's no-boundary state

- **With an ideal observer** (which can't annihilate), the boundary conditions include the observer's clock.

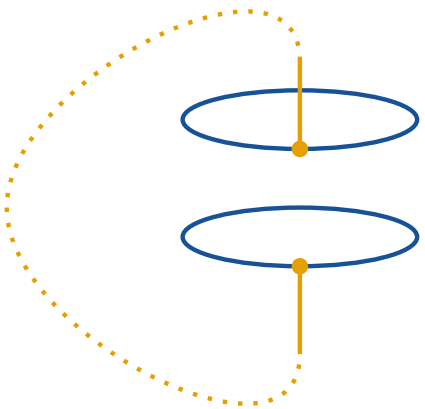
The observer's no-boundary state

- **With an ideal observer** (which can't annihilate), the boundary conditions include the observer's clock.
- For a single boundary, the observer worldline has nowhere to end:

The observer's no-boundary state

- **With an ideal observer** (which can't annihilate), the boundary conditions include the observer's clock.
- For a single boundary, the observer worldline has nowhere to end:

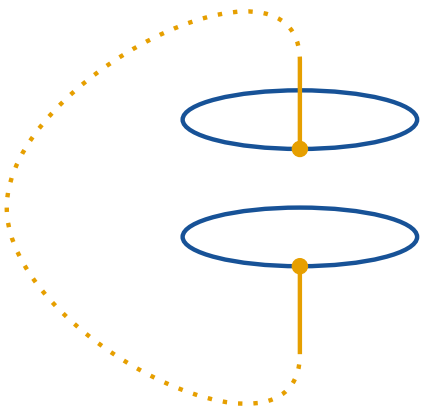
- However for **the density matrix**, $\rho_{\mathrm{NB}}(\phi_i, \phi_f)$, the observer worldline can start at ϕ_i and end at ϕ_f:



The observer's no-boundary state

- **With an ideal observer** (which can't annihilate), the boundary conditions include the observer's clock.
- For a single boundary, the observer worldline has nowhere to end:

- However for **the density matrix**, $\rho_{\text{NB}}(\phi_i, \phi_f)$, the observer worldline can start at ϕ_i and end at ϕ_f:



- Such connected geometries necessarily imply that **the observer's no-boundary state is mixed**. "bra-ket wormhole" **[Page '86, Hawking '87, Chen Gorbenko Maldacena '20,....]**.

The no-boundary state is the identity 1

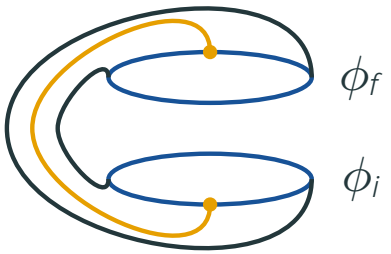
- Including an observer, the no-boundary state between ϕ_i and ϕ_f is given by manifolds $M \times I$

$$\langle \phi_f | \rho_{NB} | \phi_i \rangle = \int \mathcal{D}\phi_{\text{bulk}} \quad \phi_{\text{bulk}} \quad \phi_f \quad \phi_i \quad . \tag{4}$$

The no-boundary state is the identity 1

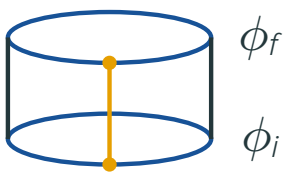
- Including an observer, the no-boundary state between ϕ_i and ϕ_f is given by manifolds $M \times I$

$$\langle \phi_f | \, \rho_{NB} \, | \phi_i \rangle = \int \mathcal{D}\phi_{\text{bulk}} \quad \phi_{\text{bulk}} \qquad . \tag{4}$$



- This is the same path-integral used to define inner-products in the Hilbert space of closed universes!

$$\langle \phi_f | \, \phi_i \rangle = \int \mathcal{D}\phi_{\text{bulk}} \qquad . \tag{5}$$

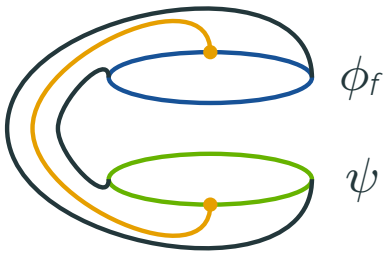


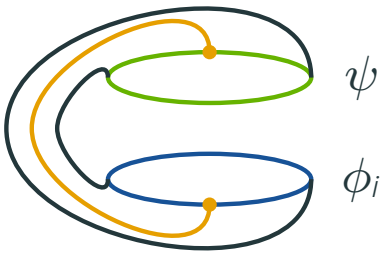
This implies that $\rho_{\text{NB}} = \mathbb{1}$.

The no-boundary state is the identity 2

- As a sanity check, we can also try to compute ρ_{NB}^2, graphically

$$\langle \phi_f | \, \rho_{\text{NB}}^2 \, | \phi_i \rangle = \int_{\text{B.C}/\text{Diff}} \mathcal{D}\psi$$



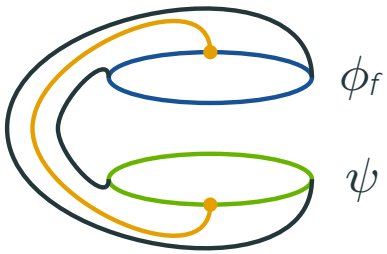


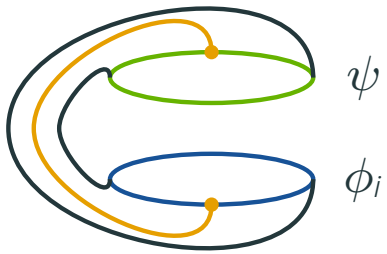
, (6)

The no-boundary state is the identity 2

- As a sanity check, we can also try to compute ρ_{NB}^2, graphically

$$\langle \phi_f | \, \rho_{\text{NB}}^2 \, | \phi_i \rangle = \int_{\text{B.C}/\text{Diff}} \mathcal{D}\psi$$





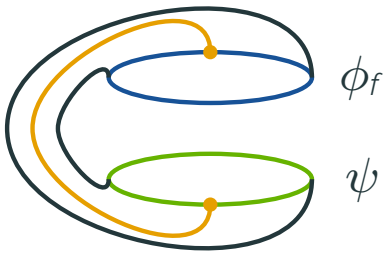
, (6)

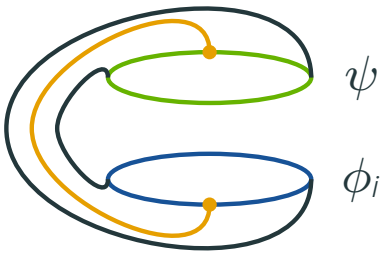
- Since the ψ integral is up to diff-invariance, this is the same as path-integrating over all $M \times I$ between ϕ_i and ϕ_f, which implies $\rho_{\text{NB}}^2 = \rho_{\text{NB}}$, and hence $\rho_{\text{NB}} = \mathbb{1}$ (unnormalized) on the space of physical states.

The no-boundary state is the identity 2

- As a sanity check, we can also try to compute ρ_{NB}^2, graphically

$$\langle \phi_f | \, \rho_{\text{NB}}^2 \, | \phi_i \rangle = \int_{\text{B.C}/\text{Diff}} \mathcal{D}\psi$$





, (6)

- Since the ψ integral is up to diff-invariance, this is the same as path-integrating over all $M \times I$ between ϕ_i and ϕ_f, which implies $\rho_{\text{NB}}^2 = \rho_{\text{NB}}$, and hence $\rho_{\text{NB}} = \mathbb{1}$ (unnormalized) on the space of physical states.
- Note that this is different from the Gibbs state $\rho_{\text{Gibbs}} = e^{-\beta H}$, although it has a similar path-integral description.

The no-boundary state is the identity 3

- Consider the ADM formalism, where the metric is

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt). \tag{7}$$

and the Einstein-Hilbert action

$$I = \int dt d^3x \left(\Pi^{ij} \dot{g}_{ij} - N H_{\mathrm{WDW}} - \sum_i N^i P_i \right). \tag{8}$$

The no-boundary state is the identity 3

- Consider the ADM formalism, where the metric is

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt). \tag{7}$$

and the Einstein-Hilbert action

$$I = \int dt d^3x \left(\Pi^{ij} \dot{g}_{ij} - N H_{\mathrm{WDW}} - \sum_i N^i P_i \right). \tag{8}$$

- For constant N minisuperspace (and $N^i = 0$), the path integral over Lorentzian $M \times I$ is

$$\langle \phi_f | \, \rho_{\mathrm{NB}} \, | \phi_i \rangle = \langle \phi_f | \, \frac{1}{2\pi} \int_{-\infty}^{+\infty} dN \, e^{-iH_{\mathrm{WDW}} N} \, | \phi_i \rangle = \langle \phi_f | \, \delta(H_{\mathrm{WDW}}) \, | \phi_i \rangle \, . \tag{9}$$

The no-boundary state is the identity 3

- Consider the ADM formalism, where the metric is

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt). \tag{7}$$

and the Einstein-Hilbert action

$$I = \int dt d^3x \left(\Pi^{ij} \dot{g}_{ij} - N H_{\text{WDW}} - \sum_i N^i P_i \right). \tag{8}$$

- For constant N minisuperspace (and $N^i = 0$), the path integral over Lorentzian $M \times I$ is

$$\langle \phi_f | \, \rho_{\text{NB}} \, | \phi_i \rangle = \langle \phi_f | \, \frac{1}{2\pi} \int_{-\infty}^{+\infty} dN \, e^{-iH_{\text{WDW}}N} \, | \phi_i \rangle = \langle \phi_f | \, \delta(H_{\text{WDW}}) \, | \phi_i \rangle \, . \tag{9}$$

- Hence $\rho_{\text{NB}} = \delta(H_{\text{WDW}})$ is the projector onto the physical states with $H_{\text{WdW}} = 0$, which is the identity on the physical Hilbert space. **[Marolf '96, Feldbrugge Lehners Turok '17, Banihashemi and T. Jacobson '25,...]**.

$\rho_{\text{NB}} = \mathbb{1}$ is automatically implemented by the Lorentzian path integral with $-\infty < N < +\infty$.

Example: closed universes in JT gravity

- JT gravity is 2d dilaton gravity with the action

$$I = \frac{1}{2}\int dx\sqrt{g}\,\Phi(R+2). \tag{10}$$

Example: closed universes in JT gravity

- JT gravity is 2d dilaton gravity with the action

$$I = \frac{1}{2}\int dx\sqrt{g}\,\Phi(R+2). \tag{10}$$

- On closed slices, the gauge-fixed variables are the dilaton Φ and the scale factor a, and their conjugate momenta Π_a, Π_Φ **[Held Maxfield '24]**.

Example: closed universes in JT gravity

- JT gravity is 2d dilaton gravity with the action

$$I = \frac{1}{2}\int dx\sqrt{g}\,\Phi(R+2). \tag{10}$$

- On closed slices, the gauge-fixed variables are the dilaton Φ and the scale factor a, and their conjugate momenta Π_a, Π_Φ **[Held Maxfield '24]**.
- The WdW Hamiltonian is $H_{wdW} = -\Pi_a\Pi_\Phi - a\Phi$. The space of physical states with $H_{\text{WdW}} = 0$ admits a complete basis:

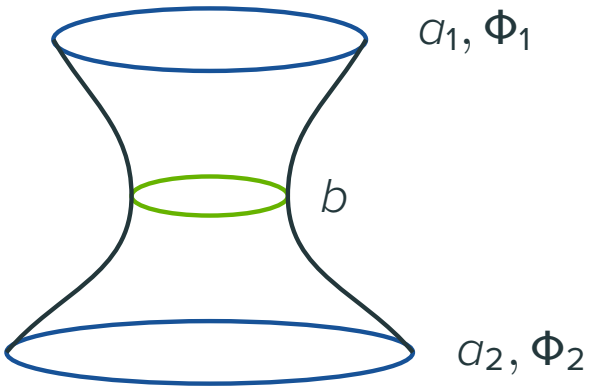
$$\langle a, \Phi | b\rangle = \frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{a^2-b^2}}e^{-\Phi\sqrt{a^2-b^2}}. \tag{11}$$

for $b \in [0,\infty)$ and $\langle b_1|b_2\rangle = \frac{1}{b_1}\delta(b_1-b_2)$.

Example: closed universes in JT gravity

- On the other hand, the no-boundary path integral gives

$$\langle a_1, \Phi_1 | \, \rho_{\text{NB}} \, | a_2, \Phi_2 \rangle =$$



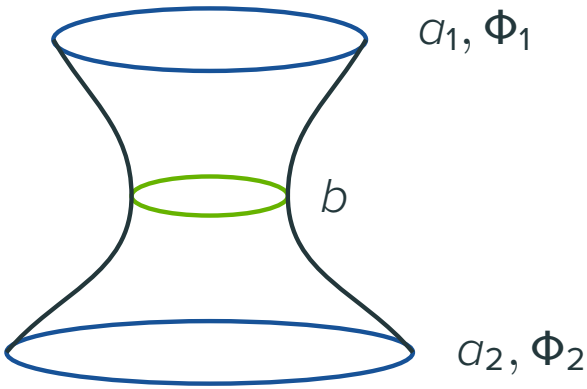
(12)

$$= \int_0^\infty b db \, \langle a_1, \Phi_1 | b \rangle \, \langle b | a_2, \Phi_2 \rangle \, . \tag{13}$$

Example: closed universes in JT gravity

- On the other hand, the no-boundary path integral gives

$$\langle a_1, \Phi_1 | \, \rho_{\text{NB}} \, | a_2, \Phi_2 \rangle = \tag{12}$$



$$= \int_0^\infty b db \, \langle a_1, \Phi_1 | b \rangle \, \langle b | a_2, \Phi_2 \rangle \, . \tag{13}$$

- Since the $|b\rangle$s form a complete basis for physical states, $\rho_{\text{NB}} = \int b db \, |b\rangle \, \langle b| = \mathbb{1}$ on the physical Hilbert space.

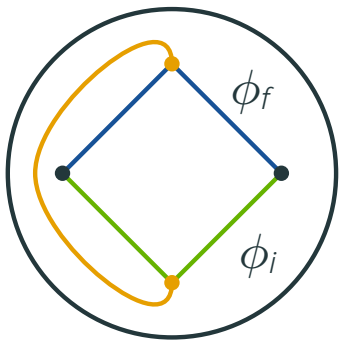
Backgrounds with cosmological horizons

For backgrounds with cosmological horizons, we follow the same prescription for the observer's no-boundary state.

Backgrounds with cosmological horizons

For backgrounds with cosmological horizons, we follow the same prescription for the observer's no-boundary state.

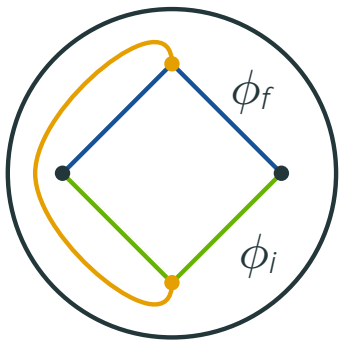
- This time, we need to specify boundary conditions for the causal diamond of the observer. The no-boundary state is given by all compact geometries that end on the observer's worldline:



Backgrounds with cosmological horizons

For backgrounds with cosmological horizons, we follow the same prescription for the observer's no-boundary state.

- This time, we need to specify boundary conditions for the causal diamond of the observer. The no-boundary state is given by all compact geometries that end on the observer's worldline:



- In de Sitter space, the no-boundary saddle is the Euclidean dS sphere, which can be thought-of as a "bra-ket wormhole" for the static patch **[CLPW '22, Ivo Li Maldacena '24]**.

Backgrounds with cosmological horizons

- By the conjecture, the entropy of the normalized no-boundary state $\tilde{\rho}_{\mathrm{NB}}$

$$S(\tilde{\rho}_{\mathrm{NB}}) \equiv -S_{\mathrm{rel}}(\tilde{\rho}_{\mathrm{NB}}||\rho_{\mathrm{NB}}) = \log \mathrm{tr}\rho_{\mathrm{NB}}. \tag{14}$$

  Whenever a **no-boundary geometry $\mathcal{M}$ semi-classically dominate the no-boundary state**, the entropy (in this sector) is given by minus the on-shell action

$$S(\tilde{\rho}_{\mathrm{NB}}) = \log \mathrm{tr}\rho_{\mathrm{NB}} = -I_{\mathcal{M}} + O(1). \tag{15}$$

  In general, this is **not the area law**.

Backgrounds with cosmological horizons

- By the conjecture, the entropy of the normalized no-boundary state $\tilde{\rho}_{\mathrm{NB}}$

$$S(\tilde{\rho}_{\mathrm{NB}}) \equiv -S_{\mathrm{rel}}(\tilde{\rho}_{\mathrm{NB}}||\rho_{\mathrm{NB}}) = \log \mathrm{tr}\rho_{\mathrm{NB}}. \tag{14}$$

  Whenever a **no-boundary geometry $\mathcal{M}$ semi-classically dominate the no-boundary state**, the entropy (in this sector) is given by minus the on-shell action

$$S(\tilde{\rho}_{\mathrm{NB}}) = \log \mathrm{tr}\rho_{\mathrm{NB}} = -I_{\mathcal{M}} + O(1). \tag{15}$$

  In general, this is **not the area law**.

- As in CLPW, we expect the algebra in those cases to be type II. However, for a general no-boundary background (lacking a timelike isometry) it is not clear how to construct it.

The $D = 3$ de Sitter axion wormhole

Axion wormholes

[Myers '88,Aguilar-Gutierrez, Hertog, Tielemans, van der Schaar, Van Riet '23]

- Consider $D = 3$ gravity with a $\Lambda > 0$ together with an axion flux density ($H = 1$) $Q = q(16\pi G)^{-1/2}$ with $0 \leq q \leq 1$. We look for an FLRW solution

$$ds^2 = -dt^2 + a(t)^2 d\Omega_2^2, \qquad d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2. \tag{16}$$

Axion wormholes

[Myers '88,Aguilar-Gutierrez, Hertog, Tielemans, van der Schaar, Van Riet '23]

- Consider $D = 3$ gravity with a $\Lambda > 0$ together with an axion flux density ($H = 1$) $Q = q(16\pi G)^{-1/2}$ with $0 \leq q \leq 1$. We look for an FLRW solution

$$ds^2 = -dt^2 + a(t)^2 d\Omega_2^2, \qquad d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2. \tag{16}$$

- The solution is

$$a^2(t) = \frac{1 + \sqrt{1-q^2}\cosh(2t)}{2}. \tag{17}$$

Axion wormholes

[Myers '88,Aguilar-Gutierrez, Hertog, Tielemans, van der Schaar, Van Riet '23]

- Consider $D = 3$ gravity with a $\Lambda > 0$ together with an axion flux density ($H = 1$) $Q = q(16\pi G)^{-1/2}$ with $0 \leq q \leq 1$. We look for an FLRW solution

$$ds^2 = -dt^2 + a(t)^2 d\Omega_2^2, \qquad d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2. \tag{16}$$

- The solution is

$$a^2(t) = \frac{1 + \sqrt{1-q^2}\cosh(2t)}{2}. \tag{17}$$

- $q = 0$: dS $a(t) = \cosh(t)$.
- $q = 1$: Einstein's static universe $a^2(t) = 1/2$.

Axion wormholes

[Myers '88,Aguilar-Gutierrez, Hertog, Tielemans, van der Schaar, Van Riet '23]

- Consider $D = 3$ gravity with a $\Lambda > 0$ together with an axion flux density ($H = 1$) $Q = q(16\pi G)^{-1/2}$ with $0 \leq q \leq 1$. We look for an FLRW solution

$$ds^2 = -dt^2 + a(t)^2 d\Omega_2^2, \qquad d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2. \tag{16}$$

- The solution is

$$a^2(t) = \frac{1 + \sqrt{1-q^2}\cosh(2t)}{2}. \tag{17}$$

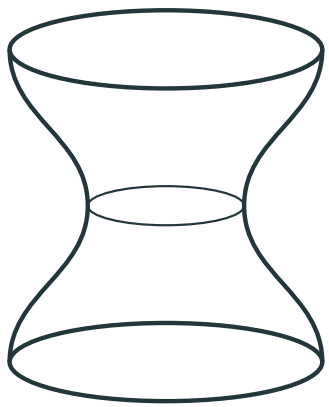
- $q = 0$: dS $a(t) = \cosh(t)$.
- $q = 1$: Einstein's static universe $a^2(t) = 1/2$.
- The solution is asymptotically de Sitter for $|t| \gg 1$.

Axion wormholes

- In Euclidean signature $t = i\tau$, the solution is

$$a^2(\tau) = \frac{1 + \sqrt{1-q^2}\cos(2\tau)}{2}. \tag{18}$$

The solution starts at $\tau = 0$ with $a^2_{\text{max}} = \frac{1+\sqrt{1-q^2}}{2}$, reaches its minimal size $a_{\text{min}} = \frac{1-\sqrt{1-q^2}}{2}$ at $\tau = \pi/2$ and then grows back to a_{max} at $\tau = \pi$:

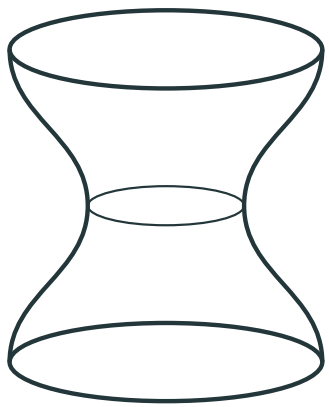


Axion wormholes

- In Euclidean signature $t = i\tau$, the solution is

$$a^2(\tau) = \frac{1 + \sqrt{1-q^2}\cos(2\tau)}{2}. \tag{18}$$

The solution starts at $\tau = 0$ with $a^2_{\text{max}} = \frac{1+\sqrt{1-q^2}}{2}$, reaches its minimal size $a_{\text{min}} = \frac{1-\sqrt{1-q^2}}{2}$ at $\tau = \pi/2$ and then grows back to a_{max} at $\tau = \pi$:



- This is a **no-boundary wormhole geometry**, with an on-shell action

$$I_1 = -\pi(1-q)/(2G). \tag{19}$$

Axion wormholes: the puzzle

- **“Necklaces”**: since the Euclidean solution is perodic $\tau \sim \tau + \pi$, we can glue k copies of the solution together:

Axion wormholes: the puzzle

- **“Necklaces”**: since the Euclidean solution is perodic $\tau \sim \tau + \pi$, we can glue k copies of the solution together:

- Taking $0 \leq \tau \leq \mathbf{N_k = \pi k}$ for integer k gives a no-boundary saddle with on-shell action $I_k = k \cdot I_1$.

Axion wormholes: the puzzle

- **“Necklaces”**: since the Euclidean solution is perodic $\tau \sim \tau + \pi$, we can glue k copies of the solution together:

- Taking $0 \leq \tau \leq \mathbf{N_k = \pi k}$ for integer k gives a no-boundary saddle with on-shell action $I_k = k \cdot I_1$.
- Since all the necklaces satisfy the same boundary conditions, in any sensible Euclidean prescription for ρ_{NB} we need to sum over all of them, which gives an unbounded answer!

$$\rho_{\text{NB}} \sim \sum_{k}^{\infty} e^{k \cdot |I_1|} = \infty. \tag{20}$$

Axion wormholes: the puzzle

- **“Necklaces”**: since the Euclidean solution is perodic $\tau \sim \tau + \pi$, we can glue k copies of the solution together:

- Taking $0 \leq \tau \leq \mathbf{N_k = \pi k}$ for integer k gives a no-boundary saddle with on-shell action $I_k = k \cdot I_1$.
- Since all the necklaces satisfy the same boundary conditions, in any sensible Euclidean prescription for $\rho_{\rm NB}$ we need to sum over all of them, which gives an unbounded answer!

$$\rho_{\rm NB} \sim \sum_{k}^{\infty} e^{k \cdot |I_1|} = \infty. \tag{20}$$

- **This is the dS axion wormhole puzzle we would like to address.**

Minisuperspace analysis

- In order to address the puzzle in a tractable way, we performed a **truncation to minisuperspace** [Halliwell Myers '89]. [Dorronsoro,Halliwell,Hartle,Hertog,Janssen '17]

Minisuperspace analysis

- In order to address the puzzle in a tractable way, we performed a **truncation to minisuperspace** [Halliwell Myers '89]. [Dorronsoro,Halliwell,Hartle,Hertog,Janssen '17]
- Concretely, we freeze the degrees of freedom of the axion (fix q), and consider a **constant lapse** N and a **homogeneous scale factor** $a(t)$:

$$ds^2 = N^2 d\tau^2 + a(\tau)^2 d\Omega_2^2. \tag{21}$$

Minisuperspace analysis

- In order to address the puzzle in a tractable way, we performed a **truncation to minisuperspace** [Halliwell Myers '89]. [Dorronsoro,Halliwell,Hartle,Hertog,Janssen '17]
- Concretely, we freeze the degrees of freedom of the axion (fix q), and consider a **constant lapse** N and a **homogeneous scale factor** $a(t)$:

$$ds^2 = N^2 d\tau^2 + a(\tau)^2 d\Omega_2^2. \tag{21}$$

- The Euclidean action

$$I_E[a(t), N] = \frac{1}{2G} \int_0^N d\tau \left(-\dot{a}^2 + a^2 - 1 + \frac{q^2}{4a^2} \right). \tag{22}$$

Minisuperspace analysis

- In order to address the puzzle in a tractable way, we performed a **truncation to minisuperspace** [Halliwell Myers '89]. [Dorronsoro,Halliwell,Hartle,Hertog,Janssen '17]
- Concretely, we freeze the degrees of freedom of the axion (fix q), and consider a **constant lapse** N and a **homogeneous scale factor** $a(t)$:

$$ds^2 = N^2 d\tau^2 + a(\tau)^2 d\Omega_2^2. \tag{21}$$

- The Euclidean action

$$I_E[a(t), N] = \frac{1}{2G} \int_0^N d\tau \left(-\dot{a}^2 + a^2 - 1 + \frac{q^2}{4a^2} \right). \tag{22}$$

- Note that the action has an $a \mapsto -a$ symmetry. Since the metric is identified under $a \mapsto -a$, we need to **gauge this symmetry**.

The no-boundary density matrix

- The matrix element of the no-boundary state between two scale factors a_1 and a_2 is given by

$$\langle a_1 |\, \rho_{\rm NB}\, | a_2 \rangle = \int_C dN\, \tilde{\mathcal{G}}(a_1, a_2; N), \tag{23}$$

The no-boundary density matrix

- The matrix element of the no-boundary state between two scale factors a_1 and a_2 is given by

$$\langle a_1 |\, \rho_{\mathrm{NB}} \,| a_2 \rangle = \int_C dN\, \tilde{\mathcal{G}}(a_1, a_2; N), \tag{23}$$

- Due to the $a \mapsto -a$ gauging, the propagator is actually a sum of two standard propagators:

$$\tilde{\mathcal{G}}(a_1, a_2; N) = \mathcal{G}(a_1, a_2; N) + \mathcal{G}(a_1, -a_2; N), \tag{24}$$

The no-boundary density matrix

- The matrix element of the no-boundary state between two scale factors a_1 and a_2 is given by

$$\langle a_1 | \rho_{\rm NB} | a_2 \rangle = \int_C dN \, \tilde{\mathcal{G}}(a_1, a_2; N), \tag{23}$$

- Due to the $a \mapsto -a$ gauging, the propagator is actually a sum of two standard propagators:

$$\tilde{\mathcal{G}}(a_1, a_2; N) = \mathcal{G}(a_1, a_2; N) + \mathcal{G}(a_1, -a_2; N), \tag{24}$$

- Each propagator satisfies $\nu = \sqrt{\frac{1}{4} - \frac{q^2}{4G^2}}$

$$\mathcal{G}(a_1, a_2; N) = \int_{a(0)=a_1}^{a(N)=a_2} \mathcal{D}a(\tau) \, \exp\left(-I_E[a(\tau), N]\right) \tag{25}$$

$$= \frac{2\sqrt{-a_1 a_2}}{\sin(N)} \exp\left\{ \frac{N}{2G} + \frac{(a_1^2 + a_2^2)}{2G \tan(N)} \right\} K_\nu \left(\frac{a_1 a_2}{G \sin(N)} \right). \tag{26}$$

The semiclassical limit

$$\tilde{\mathcal{G}}(a_1, a_2; N) = \mathcal{G}(a_1, a_2; N) + \mathcal{G}(a_1, -a_2; N) \tag{27}$$

- In the semiclassical limit $G \ll 1$, each propagator $\mathcal{G}_\pm$ is dominated by a classical solutions $a_{\pm,N}(t)$.

The semiclassical limit

$$\tilde{\mathcal{G}}(a_1, a_2; N) = \mathcal{G}(a_1, a_2; N) + \mathcal{G}(a_1, -a_2; N) \tag{27}$$

- In the semiclassical limit $G \ll 1$, each propagator $\mathcal{G}_\pm$ is dominated by a classical solutions $a_{\pm,N}(t)$.
- For simplicity (and I'm probably out of time)

The semiclassical limit

$$\tilde{\mathcal{G}}(a_1, a_2; N) = \mathcal{G}(a_1, a_2; N) + \mathcal{G}(a_1, -a_2; N) \tag{27}$$

- In the semiclassical limit $G \ll 1$, each propagator $\mathcal{G}_\pm$ is dominated by a classical solutions $a_{\pm,N}(t)$.
- For simplicity (and I'm probably out of time)
  1. Take the smooth pure gravity limit $q \to 0$. In this limit the $a = 0$ point is a "brick wall" due to the flux potential.

The semiclassical limit

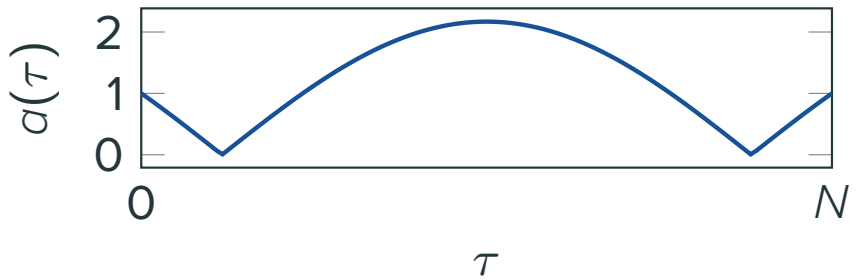
$$\tilde{\mathcal{G}}(a_1, a_2; N) = \mathcal{G}(a_1, a_2; N) + \mathcal{G}(a_1, -a_2; N) \tag{27}$$

- In the semiclassical limit $G \ll 1$, each propagator $\mathcal{G}_\pm$ is dominated by a classical solutions $a_{\pm,N}(t)$.
- For simplicity (and I'm probably out of time)
  1. Take the smooth pure gravity limit $q \to 0$. In this limit the $a = 0$ point is a "brick wall" due to the flux potential.
  2. Set $a_1 = a_2 = 1$.

The + sector

- The $a_1 \mapsto a_2$ sector is dominated in the $G \ll 1$ limit by the solution ($0 \leq \tau \leq N$)

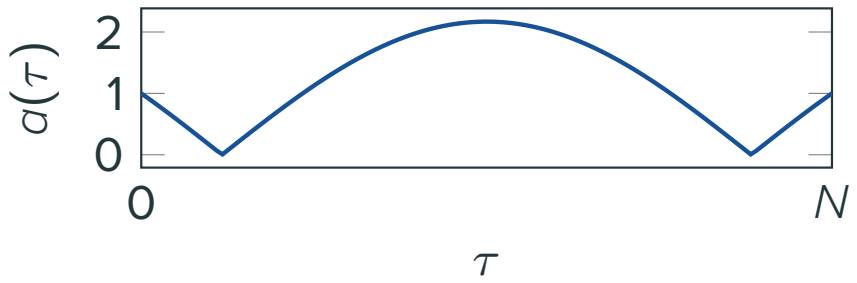
$$a(\tau) = \left| \frac{\cos(N/2 - \tau)}{\cos(N/2)} \right| . \tag{28}$$



The + sector

- The $a_1 \mapsto a_2$ sector is dominated in the $G \ll 1$ limit by the solution ($0 \leq \tau \leq N$)

$$a(\tau) = \left| \frac{\cos(N/2 - \tau)}{\cos(N/2)} \right| . \tag{28}$$

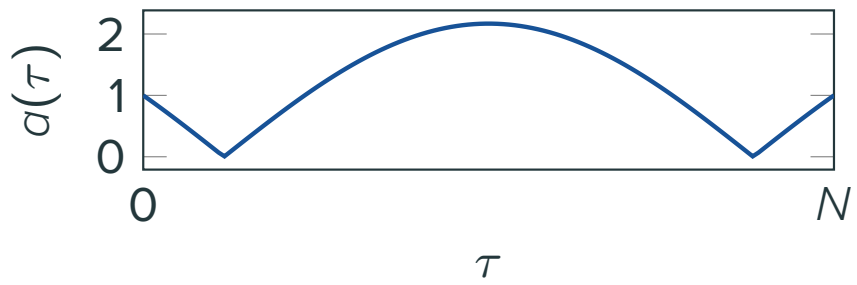


- $N_{2m} = 2m \cdot \pi$, is the **(2m)-necklace**.
  The solution **diverges** for odd $N_{2m+1} = (2m+1) \cdot \pi$.

The + sector

- The $a_1 \mapsto a_2$ sector is dominated in the $G \ll 1$ limit by the solution ($0 \leq \tau \leq N$)

$$a(\tau) = \left| \frac{\cos(N/2 - \tau)}{\cos(N/2)} \right| . \tag{28}$$



- $N_{2m} = 2m \cdot \pi$, is the **(2m)-necklace**.
  The solution **diverges** for odd $N_{2m+1} = (2m+1) \cdot \pi$.
- The on-shell action is

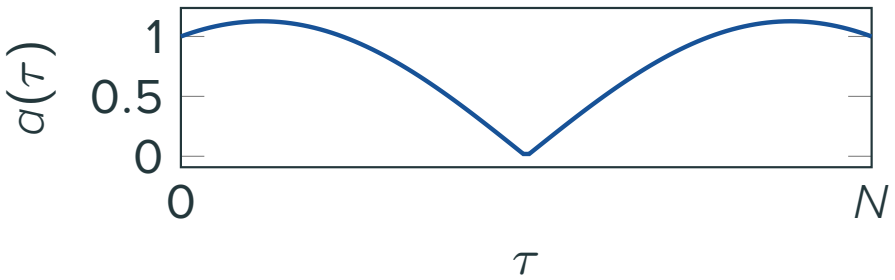
$$I^{(+)}(N) = \frac{1}{2G} \left(-N + 2 \tan(N/2) \right) . \tag{29}$$

  N_{2m} are saddles of the action with $I = 2m \cdot I_1$, while N_{2m+1} are poles of $I^{(+)}(N)$.

The $-$ sector

- The $a_1 \mapsto -a_2$ sector is dominated in the $G \ll 1$ limit by the solution ($0 \leq \tau \leq N$)

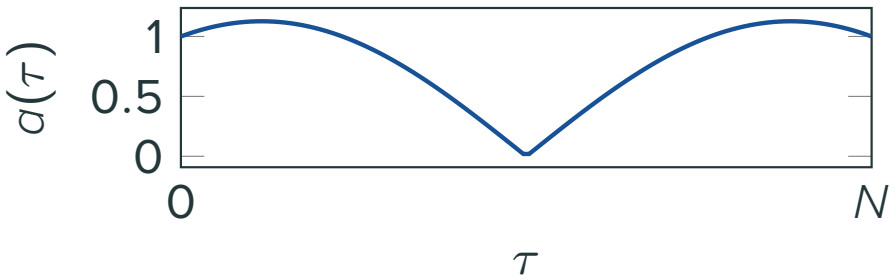
$$a(\tau) = \left| \frac{\sin(N/2 - \tau)}{\sin(N/2)} \right| . \tag{30}$$



The $-$ sector

- The $a_1 \mapsto -a_2$ sector is dominated in the $G \ll 1$ limit by the solution ($0 \leq \tau \leq N$)

$$a(\tau) = \left| \frac{\sin(N/2 - \tau)}{\sin(N/2)} \right|. \tag{30}$$

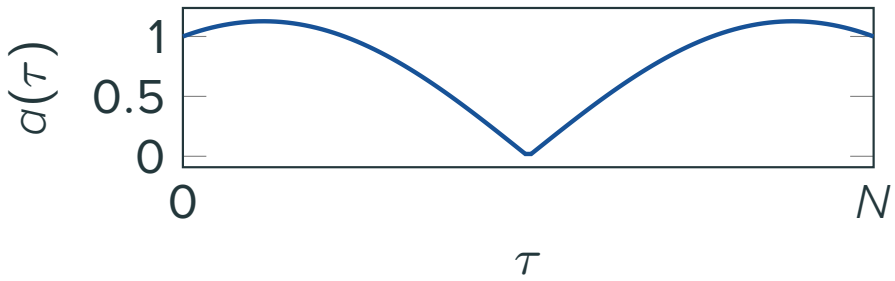


- $N_{2m+1} = (2m+1) \cdot \pi$ is the $(2m+1)$-**necklace**.
  The solution **divrges** for even $N_{2m} = 2m \cdot \pi$.

The $-$ sector

- The $a_1 \mapsto -a_2$ sector is dominated in the $G \ll 1$ limit by the solution ($0 \leq \tau \leq N$)

$$a(\tau) = \left| \frac{\sin(N/2 - \tau)}{\sin(N/2)} \right| . \tag{30}$$



- $N_{2m+1} = (2m+1) \cdot \pi$ is the $(2m+1)$-**necklace**.
  The solution **divrges** for even $N_{2m} = 2m \cdot \pi$.
- The on-shell action is

$$I^{(-)}(N) = \frac{1}{2G} \left(-N - 2\cot(N/2) \right) . \tag{31}$$

  N_{2m+1} are saddles of the action with $I = (2m+1) \cdot I_1$, while N_{2m} are poles of $I^{(-)}(N)$.

Specifically, $I^{(-)}(N)$ has a pole at $N = 0$! ($k = 0$)

The lapse contour

- In the semiclassical limit, we found

$$\langle a_1 | \, \rho_{\mathrm{NB}} \, | a_2 \rangle \sim \int_{\mathcal{C}} dN \left(e^{-I^{(+)}(N)} + e^{-I^{(-)}(N)} \right) . \tag{32}$$

The lapse contour

- In the semiclassical limit, we found

$$\langle a_1 | \rho_{\mathrm{NB}} | a_2 \rangle \sim \int_{\mathcal{C}} dN \left(e^{-I^{(+)}(N)} + e^{-I^{(-)}(N)} \right) . \tag{32}$$

All the k-necklaces appear as saddles of either $I^{(\pm)}$.

The lapse contour

- In the semiclassical limit, we found

$$\langle a_1 | \rho_{\text{NB}} | a_2 \rangle \sim \int_{\mathcal{C}} dN \left(e^{-I^{(+)}(N)} + e^{-I^{(-)}(N)} \right) . \tag{32}$$

All the k-necklaces appear as saddles of either $I^{(\pm)}$.

- **What is the correct lapse contour $\mathcal{C}$?**
  We would like to take the pure Lorentzian contour $\mathcal{C} = i\mathbb{R}$. However, the action has a pole on the contour at $N = 0$!

The lapse contour

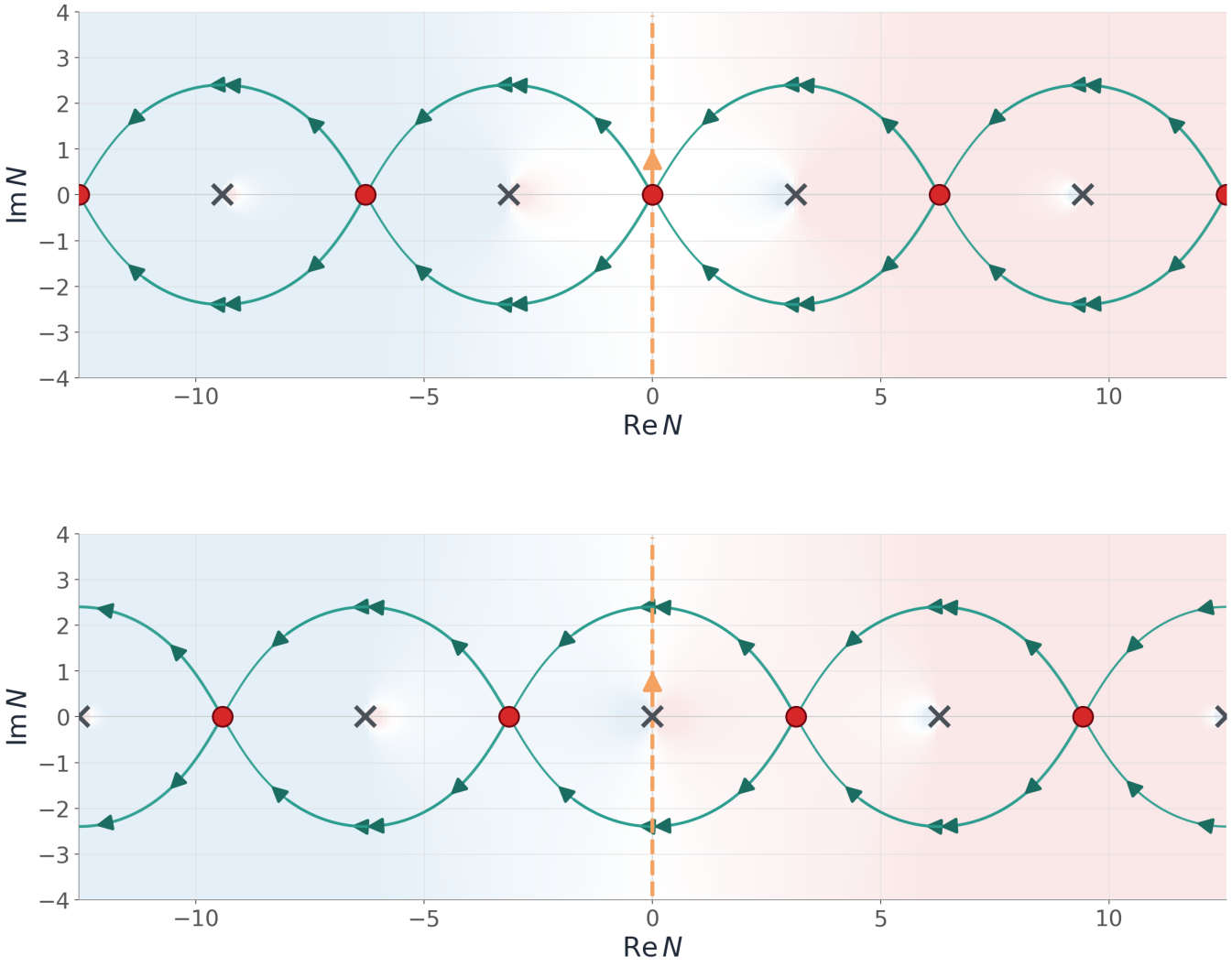
- In the semiclassical limit, we found

$$\langle a_1 | \rho_{\text{NB}} | a_2 \rangle \sim \int_{\mathcal{C}} dN \left(e^{-I^{(+)}(N)} + e^{-I^{(-)}(N)} \right) . \tag{32}$$

All the k-necklaces appear as saddles of either $I^{(\pm)}$.

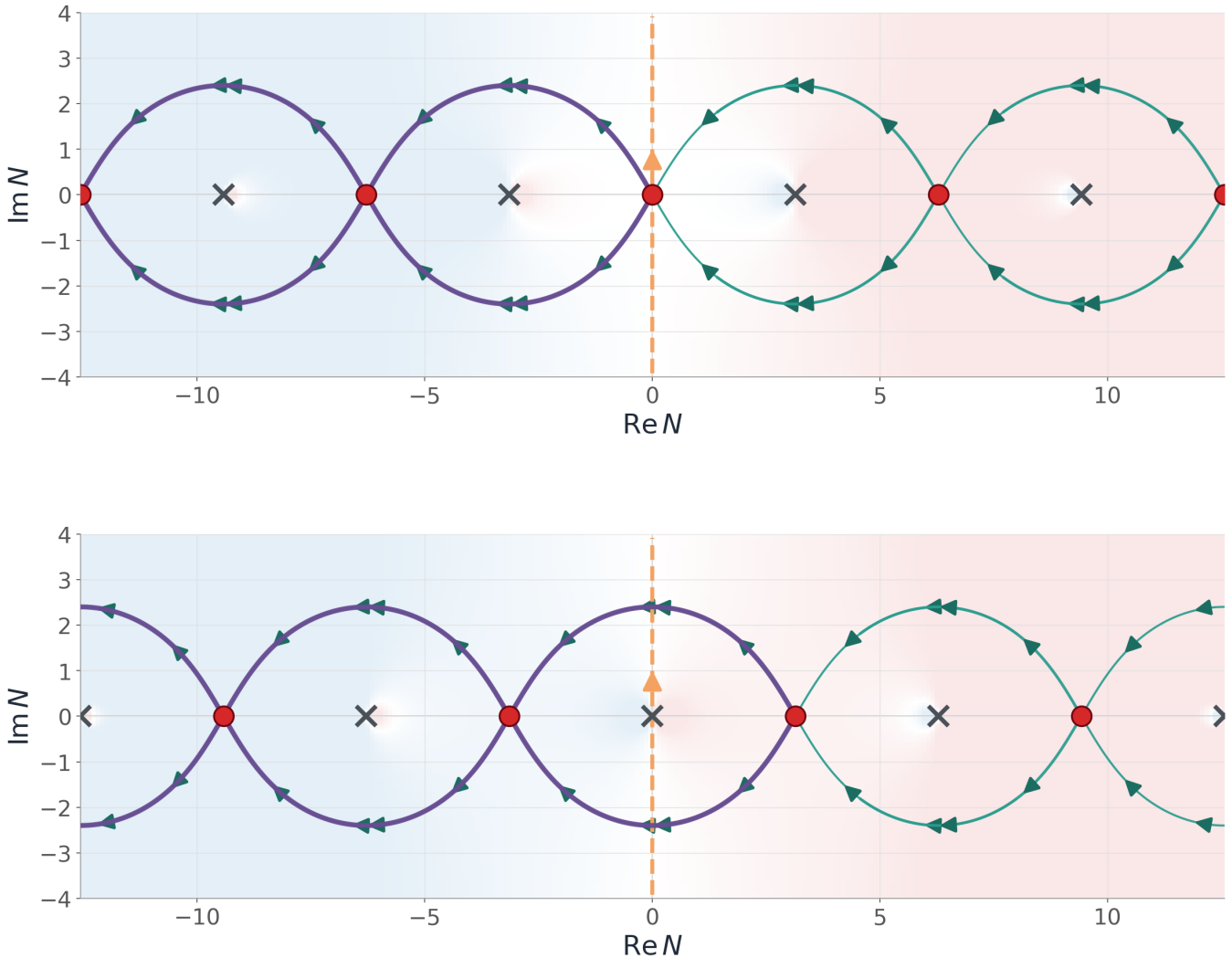
- **What is the correct lapse contour $\mathcal{C}$?**
  We would like to take the pure Lorentzian contour $\mathcal{C} = i\mathbb{R}$. However, the action has a pole on the contour at $N = 0$!
- Two options **[Hartle, Hawking, Vilenkin, Halliwell Feldbrugge, Lehners, Turok...]**:
  1. Deform to the right $\mathcal{C} = i\,\mathbb{R} + \varepsilon$ - "Hartle-Hawking"
     Similar to the $i\,\varepsilon$ prescription in QFT!
  2. Deform to the left $\mathcal{C} = i\,\mathbb{R} - \varepsilon$ - "Vilenkin"
     Treats gravitational saddles as instantons.

Steepest descent analysis: + (top) and − (bottom) sectors



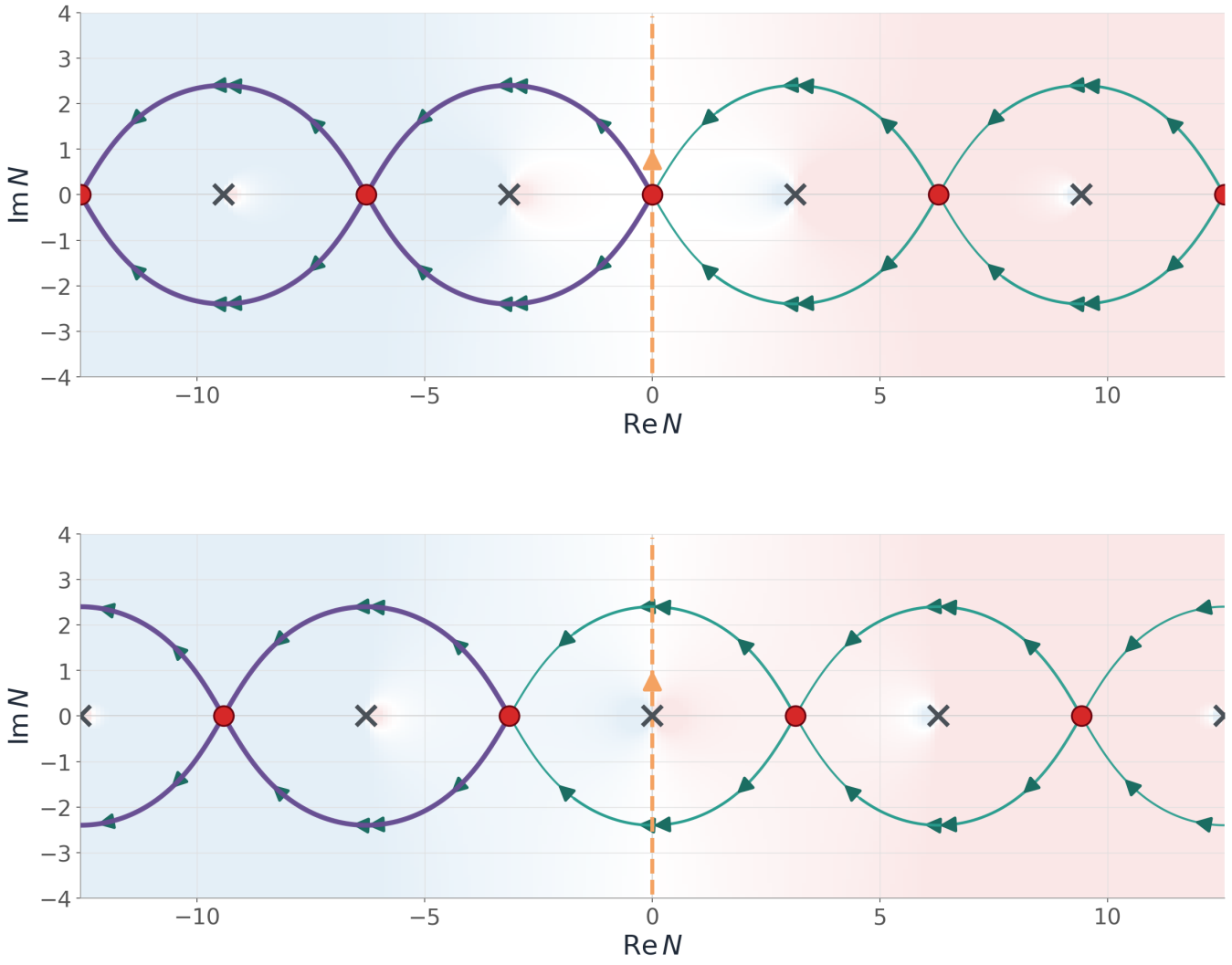
Arrows in the direction that decreases $\mathrm{Re}\, I(N)$. Saddle in red, poles in "x".

Deform to the right $\mathcal{C} = i\mathbb{R} + \varepsilon$



Purple is the steepest descent contour. Collect only N_k saddles with $k \leq 1$!

Deform to the left $\mathcal{C} = i\mathbb{R} - \varepsilon$



Purple is the steepest descent contour. Collect only N_k saddles with $k \leq 0$!

Steepest descent analysis

- For both choices of a Lorentzian contour, the deformation to steepest descent lines avoids the divergent sum over large $k \gg 1$ necklaces!
  The Lorentzian contour avoids the divergent sum!

Steepest descent analysis

- For both choices of a Lorentzian contour, the deformation to steepest descent lines avoids the divergent sum over large $k \gg 1$ necklaces!
  The Lorentzian contour avoids the divergent sum!
- When we **deform to the right** $\mathcal{C} = i\,\mathbb{R} + \varepsilon$, the steepest descent contour picks the leading $k = 1$ saddle with a negative action.
  Gives tree-level entropy

$$S = -I_1 = \pi(1-q)/2G + \ldots. \tag{33}$$

  In the dS $q = 0$ limit, we recover the area law.

Steepest descent analysis

- For both choices of a Lorentzian contour, the deformation to steepest descent lines avoids the divergent sum over large $k \gg 1$ necklaces!
  The Lorentzian contour avoids the divergent sum!
- When we **deform to the right** $\mathcal{C} = i\,\mathbb{R} + \varepsilon$, the steepest descent contour picks the leading $k = 1$ saddle with a negative action.
  Gives tree-level entropy

$$S = -I_1 = \pi(1-q)/2G + \ldots. \tag{33}$$

  In the dS $q = 0$ limit, we recover the area law.
- When we **deform to the left** $\mathcal{C} = i\,\mathbb{R} - \varepsilon$, the steepest descent contour picks, to leading order, the $k = 0$ saddles with zero action and thus entropy

$$S \sim O(1). \tag{34}$$

Discussion

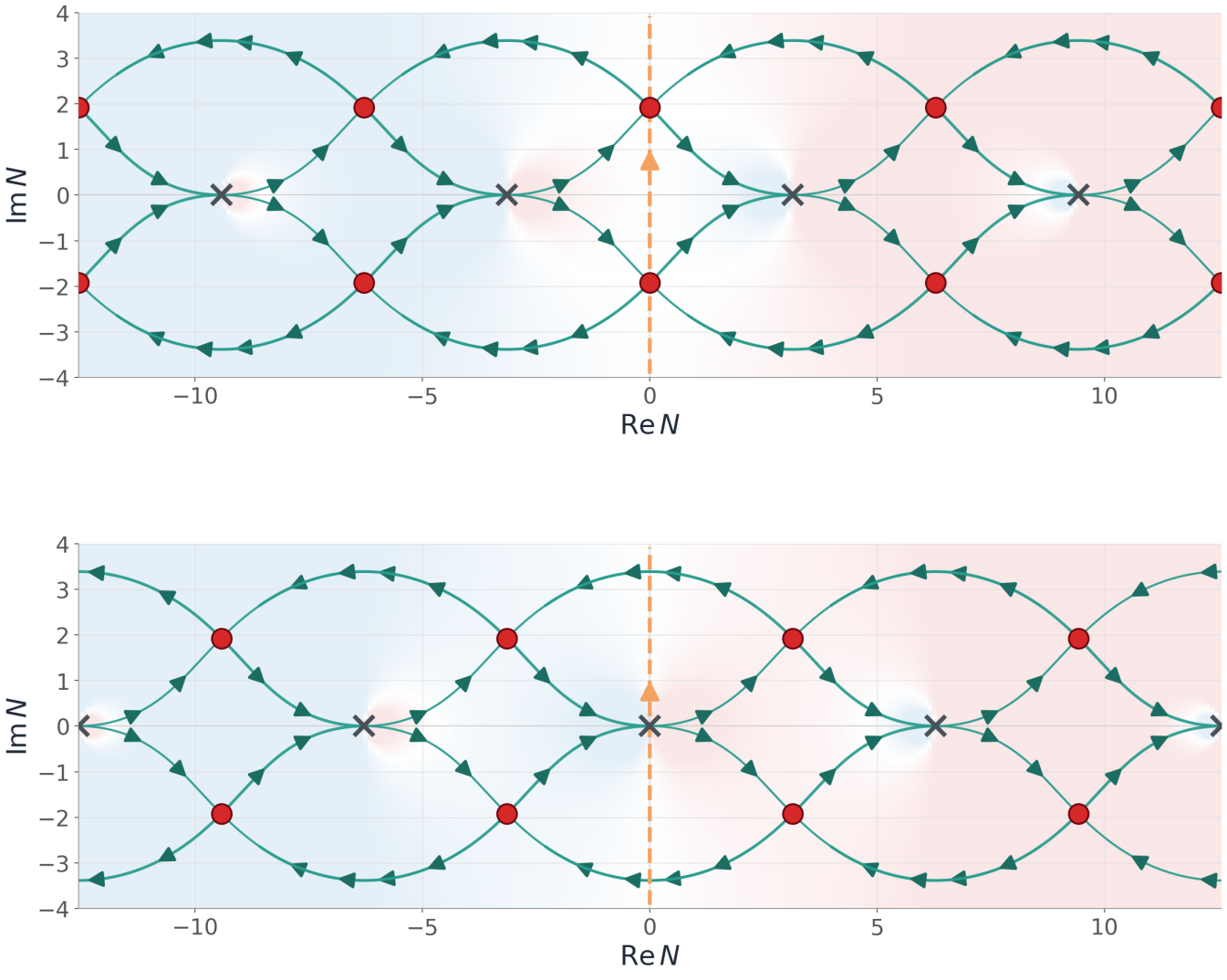
Summary

1. Provided a **path-integral evidence** for the $\rho_{\text{NB}} = \mathbb{1}$ conjecture.
2. Consistent with CLPW for dS and $\Lambda < 0$ JT gravity.
3. Whenever semiclassical approximations apply, suggests "**entropy = -action**".
4. Suggest that the no-boundary state is defined via a Lorentzian prescription.
5. Such prescription solves the "necklace puzzle" for $D = 3$ axion wormholes.

Future directions

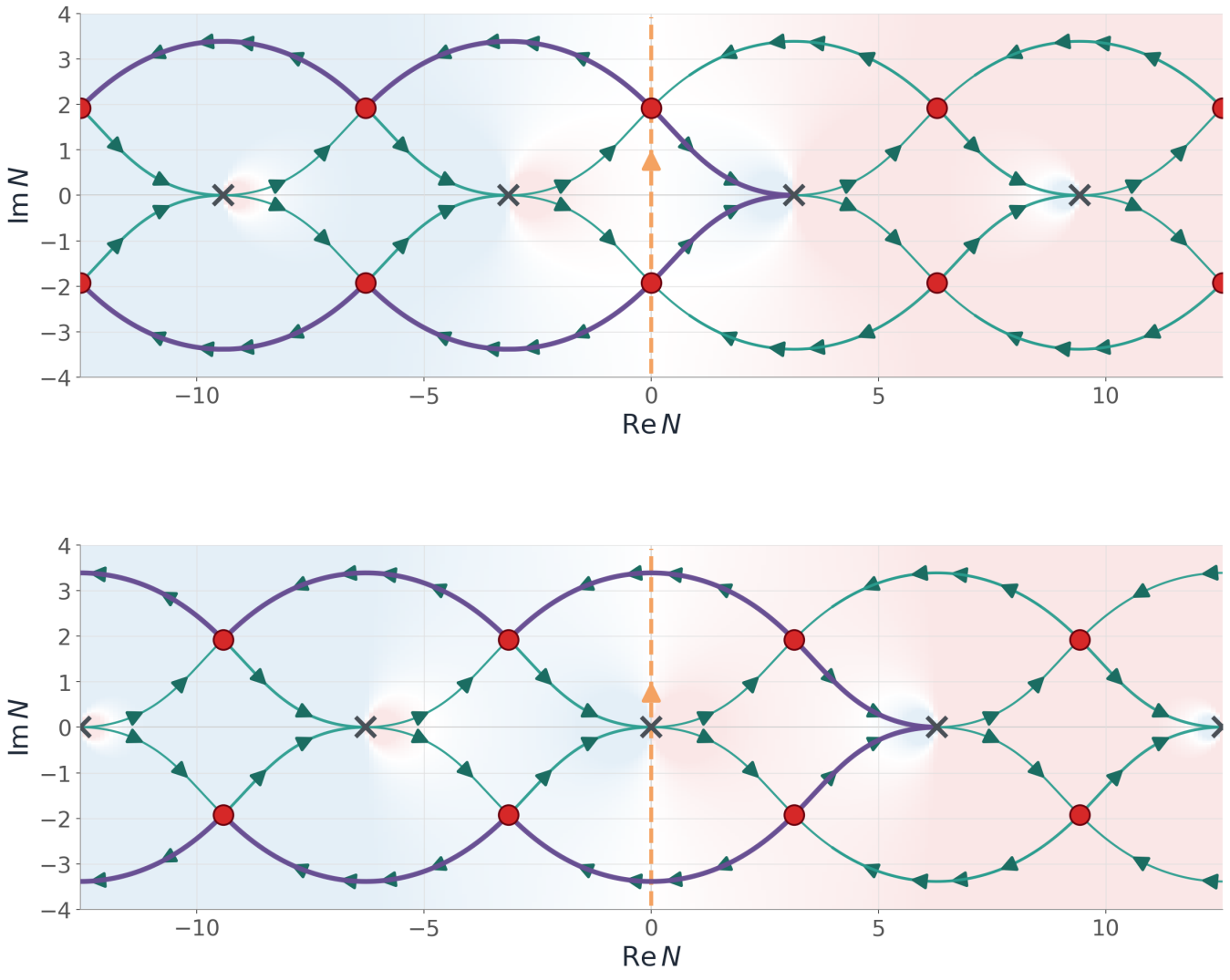
1. Other entropic puzzles?
2. How to see the transition from type II at $q = 0$ to type I at $q = 1$?
3. How to build the observer's type II algebra for other asymptotically dS backgrounds? **[Speranza, in 1m]**
4. Add quantum corrections, a less ideal clock? **[Kolchmeyer, afternoon]**

Steepest descent analysis: + (top) and − (bottom) sectors



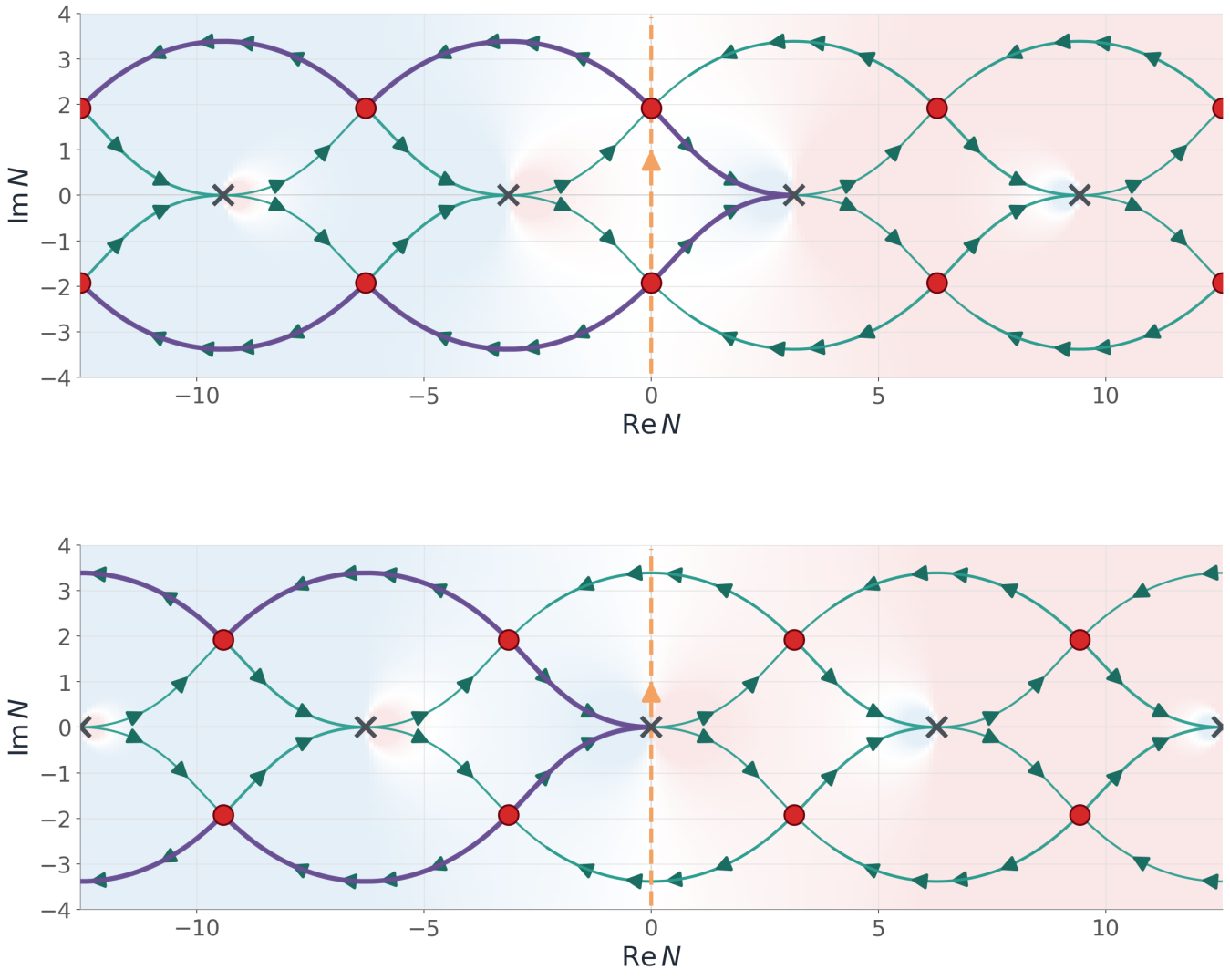
Arrows in the direction that decreases $\mathrm{Re}\, I(N)$. Saddle in red, poles in “x”.

Deform to the right $\mathcal{C} = i\mathbb{R} + \varepsilon$



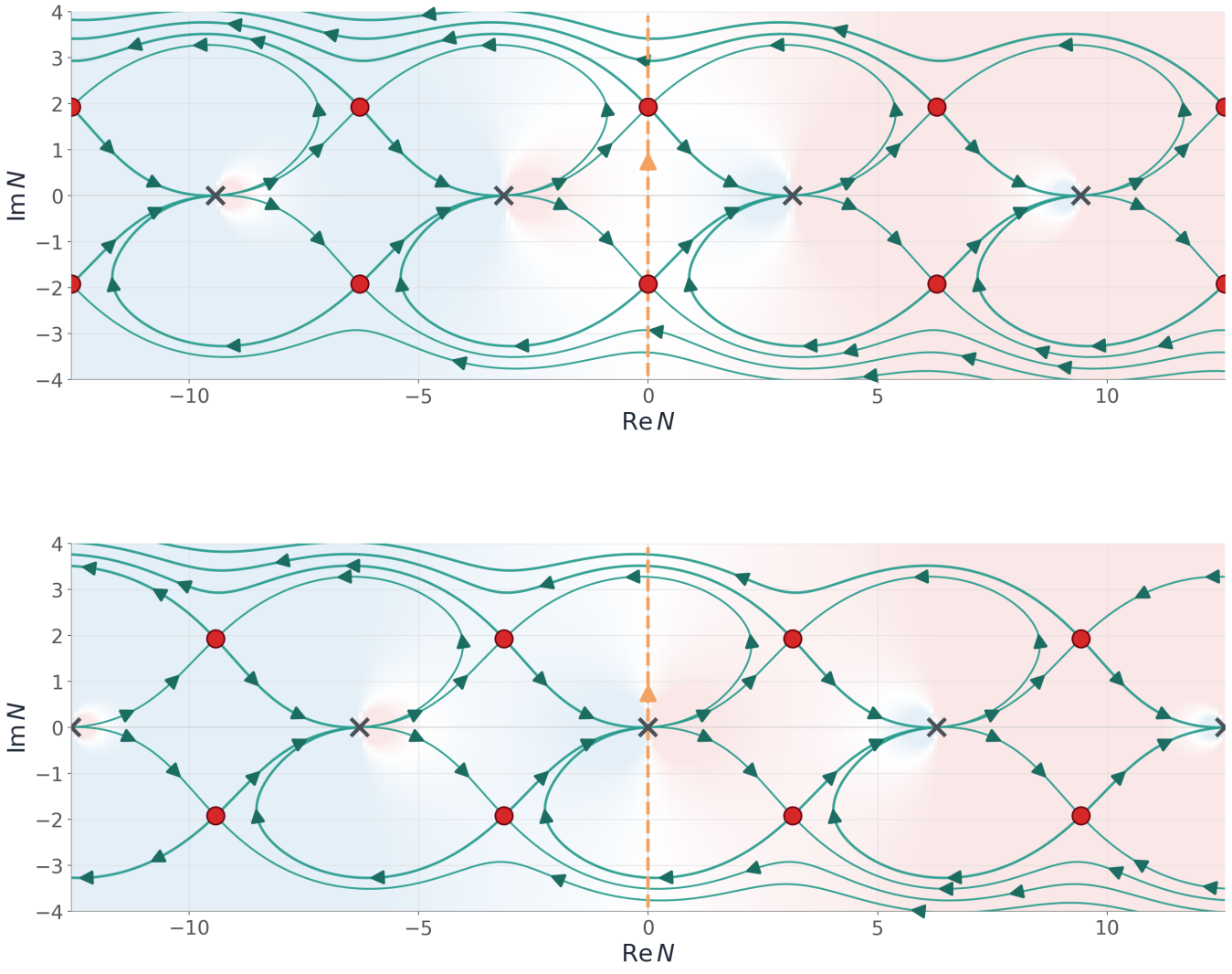
Purple is the steepest descent contour. Collect only N_k saddles with $k \leq 1$!

Deform to the left $\mathcal{C} = i\mathbb{R} - \varepsilon$



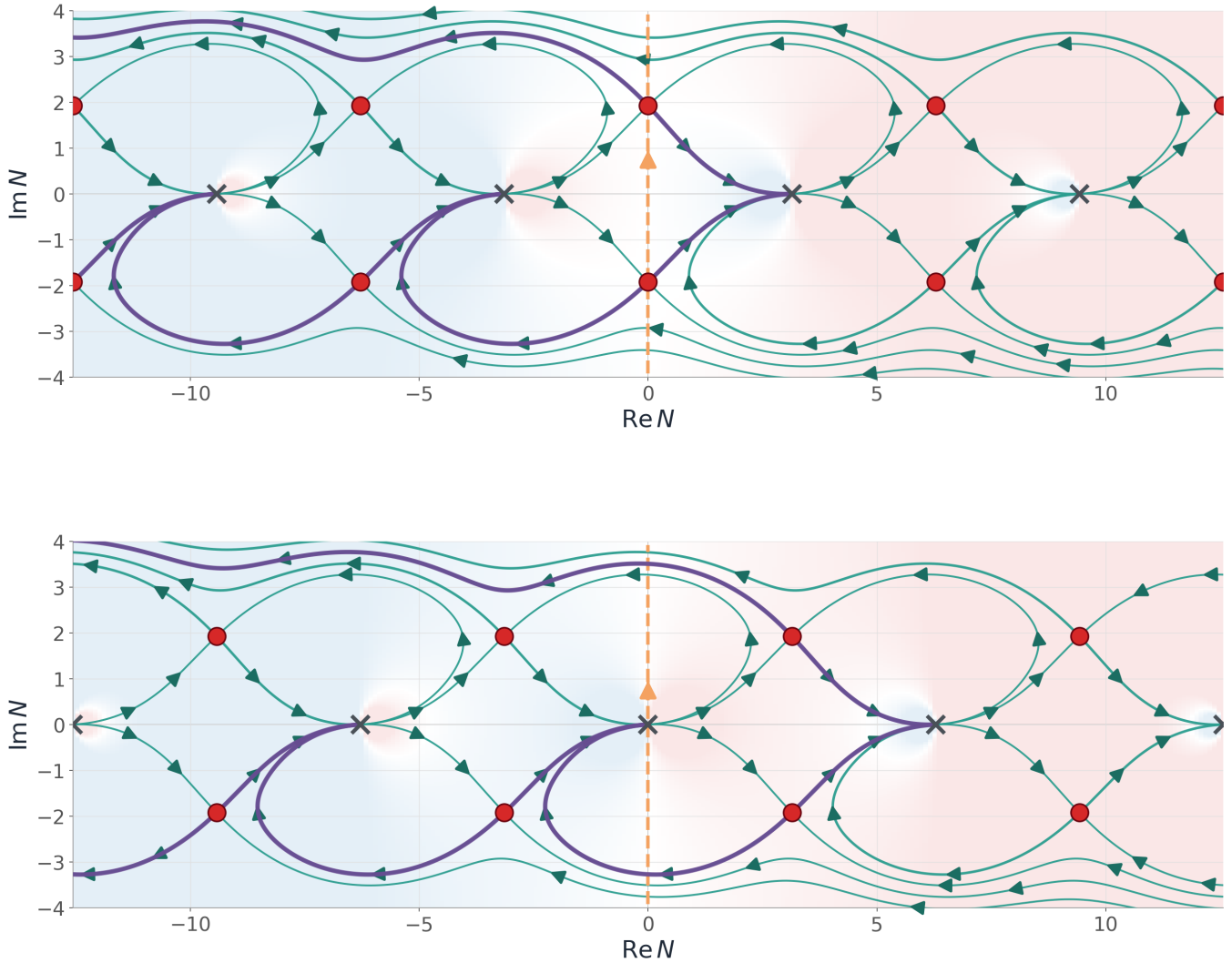
Purple is the steepest descent contour. Collect only N_k saddles with $k \leq 0$!

Steepest descent analysis with ε: $+$ (top) and $-$ (bottom) sectors



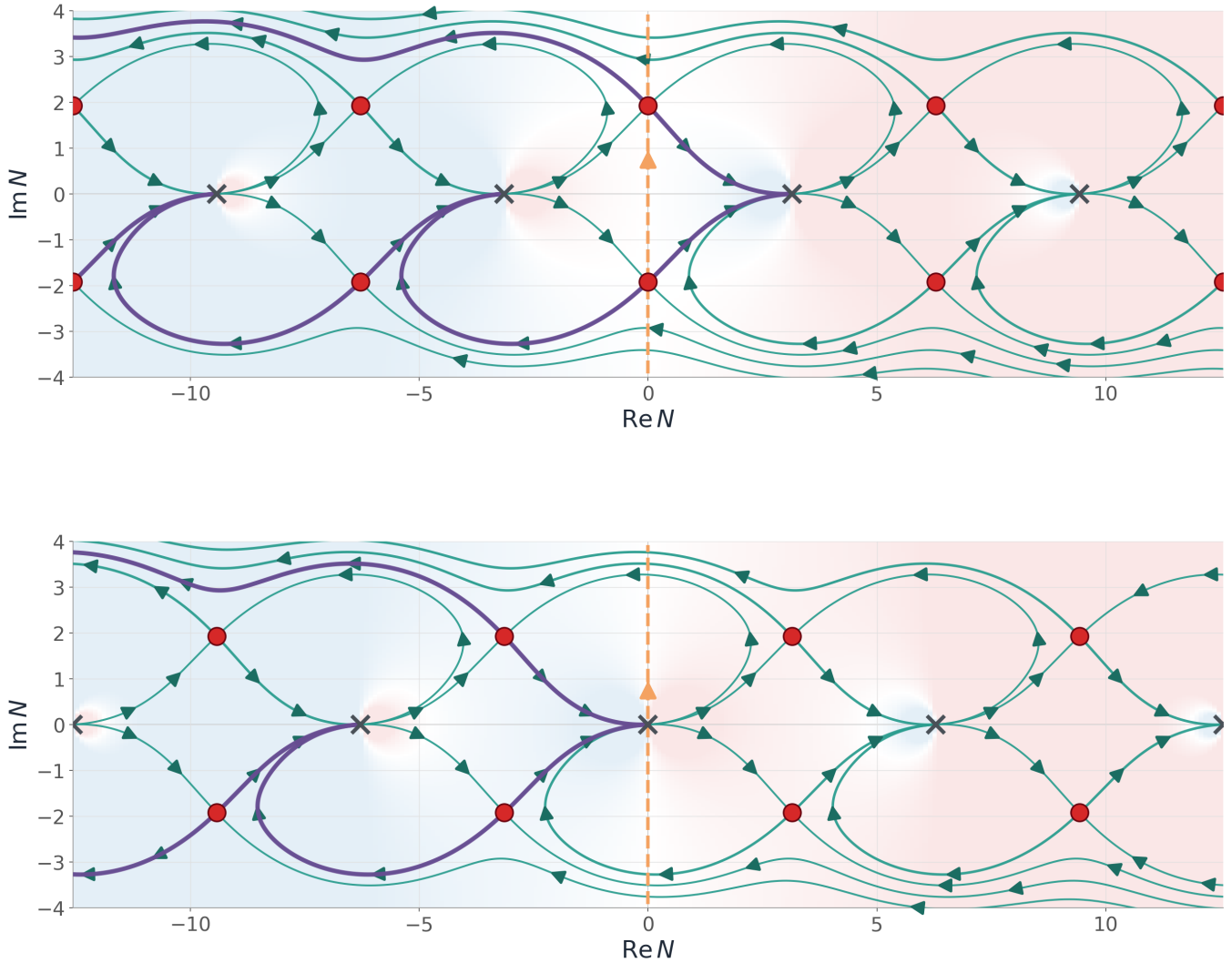
Arrows in the direction that decreases $\mathrm{Re}\, I(N)$. Saddle in red, poles in “x”.

With ε: deform to the right



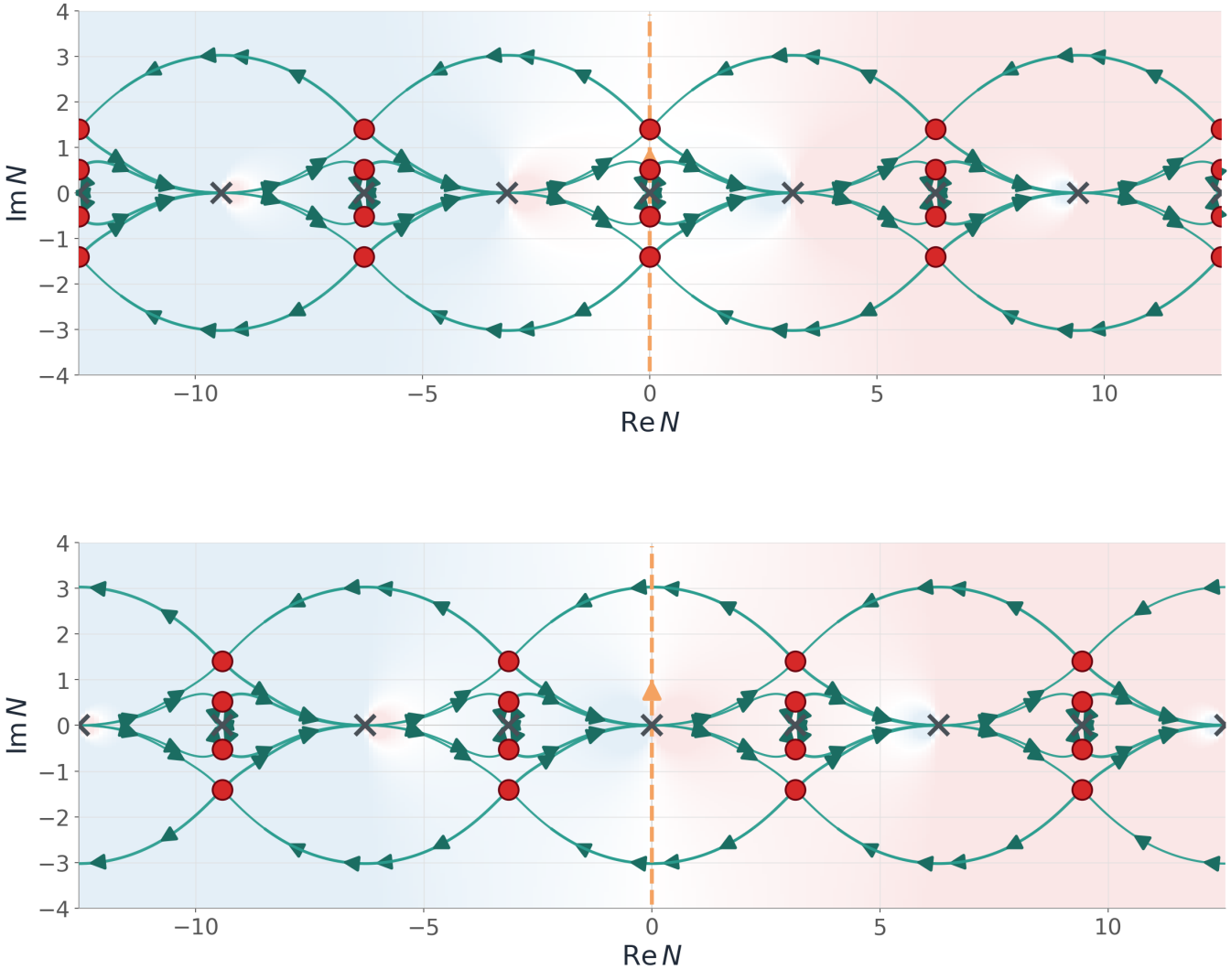
Purple is the steepest descent contour. Collect only N_k saddles with $k \leq 1$!

With ε: deform to the left



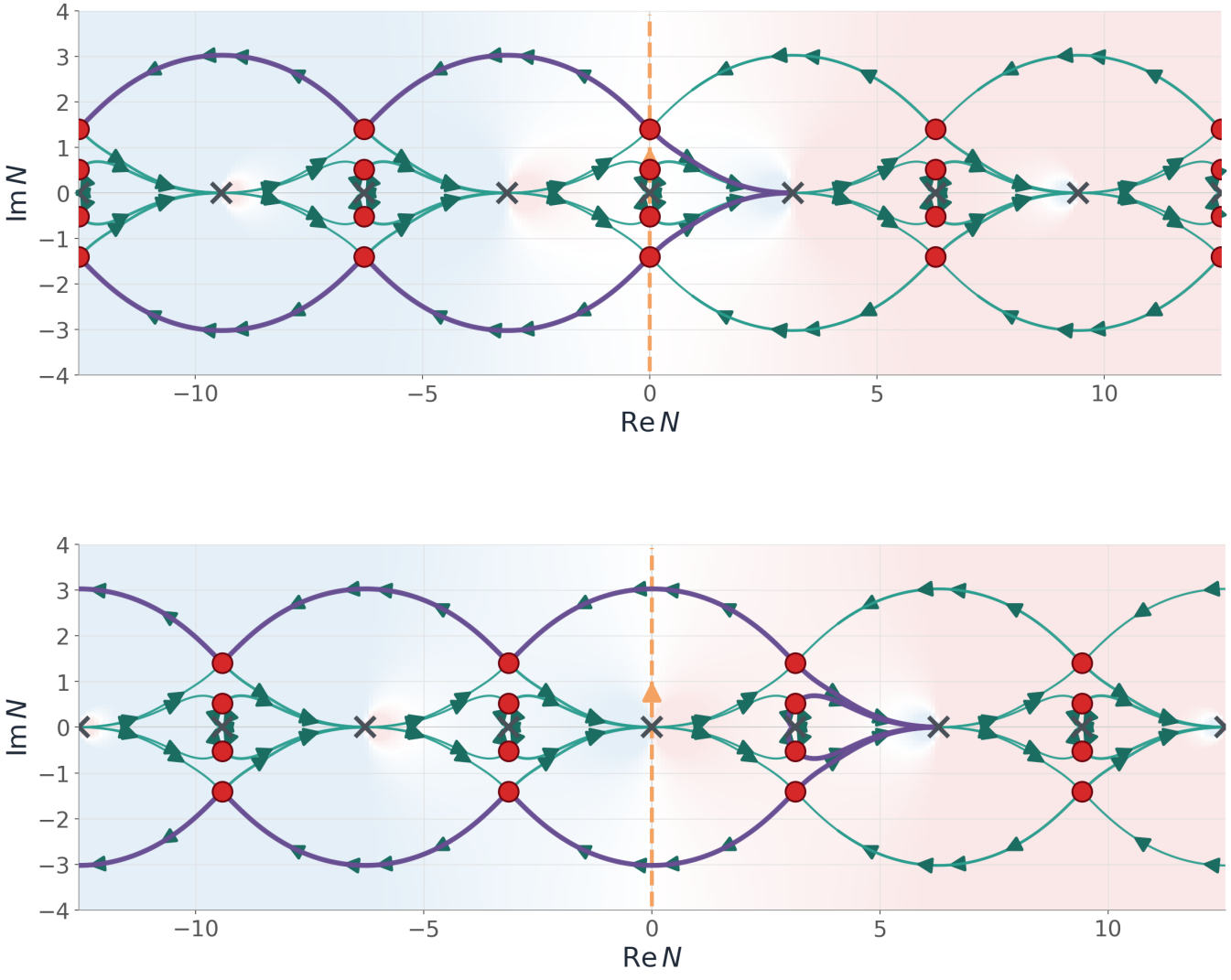
Purple is the steepest descent contour. Collect only N_k saddles with $k \leq 0$!

Steepest descent analysis, off-diagonal: + (top) and − (bottom) sectors



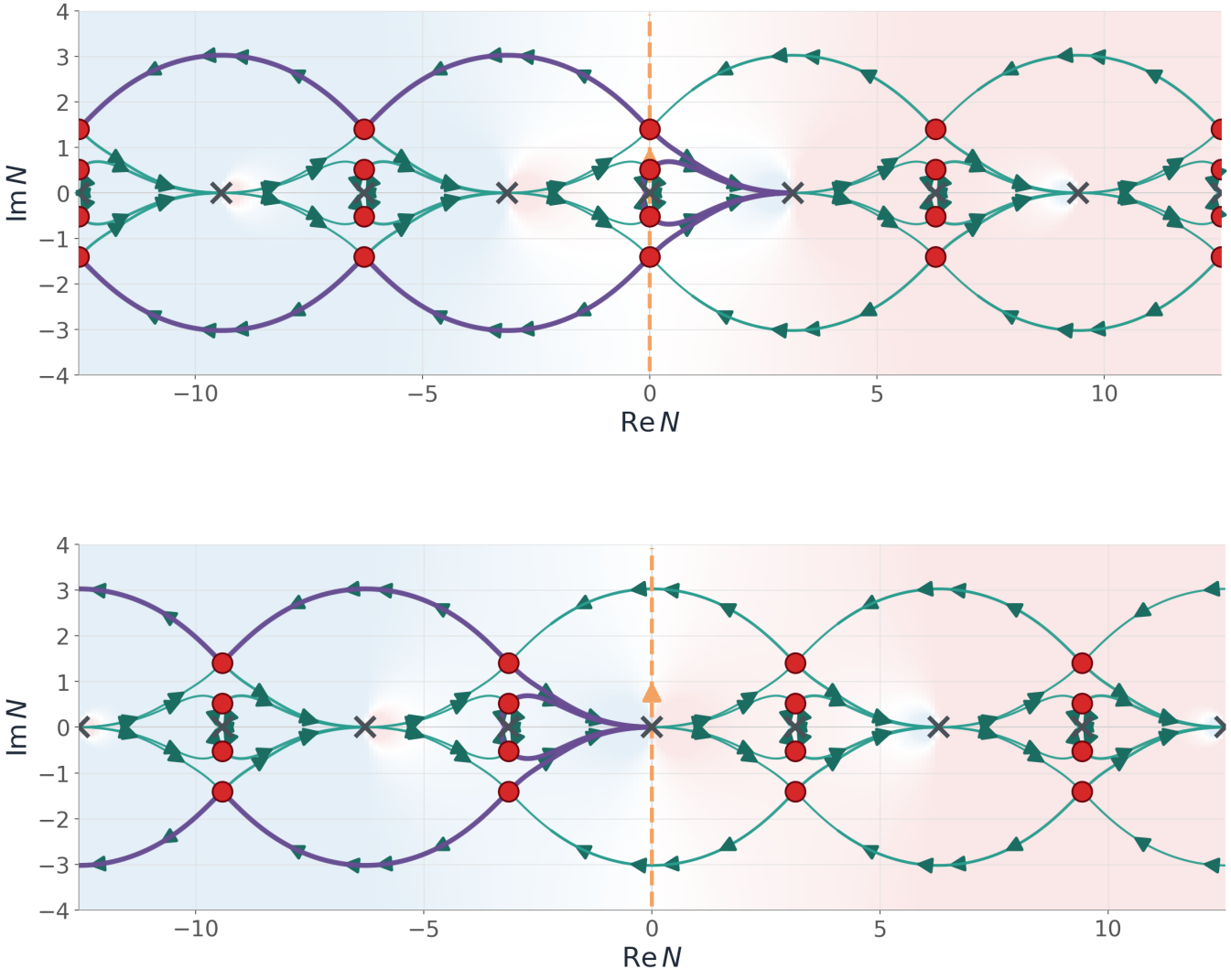
Arrows in the direction that decreases $\mathrm{Re}\, I(N)$. Saddle in red, poles in "x".

Off-diagonal: deform to the right



Purple is the steepest descent contour. Collect only N_k saddles with $k \leq 1$!

Off-diagonal: deform to the left



Purple is the steepest descent contour. Collect only N_k saddles with $k \leq 0$!