

Algebraic observers and nonequilibrium backgrounds

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Observers, Wormholes, and Complex Saddles in Cosmology
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Semiclassical and Semifinite

Semiclassical gravity in $G \rightarrow 0$ limit is qualitatively different from QFT:

- ▶ QFT in CST plus a constraint (diff invariance)
- ▶ Subregion algebras are **semifinite**: possess a trace τ

$$\tau(ab) = \tau(ba)$$

Algebras are **Type II** von Neumann algebras

- ▶ Leads to UV-finite density matrices and entropies:

$$\langle \psi | a | \psi \rangle = \tau(\rho_\psi a); \quad S_\psi = -\tau(\rho_\psi \log \rho_\psi)$$

S_ψ is known as **Segal entropy** (not von Neumann entropy!) since it can be negative; interpreted as entropy difference

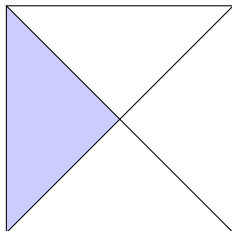
- ▶ Nontrivial matches to generalized entropy:

$$S_\psi = \left\langle \frac{A}{4G} \right\rangle_\psi + S_{\text{QFT}}^{\text{vN}} - \text{const.}$$

Example: observer dS in static patch

CLPW [Chandrasekaran, Longo, Penington, Witten 2022] model involves the static patch of de Sitter space. Algebra is obtained by gauging the time-translation symmetry in the patch.

With ordinary fields, leads to trivial algebra \rightarrow time translation acts ergodically

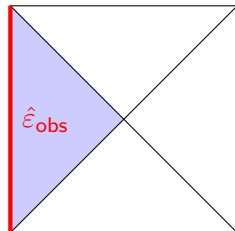


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CLPW obtain a nontrivial algebra by introducing a gravitating **observer** with Hamiltonian \hat{E}_{obs} .



Morally observer provides feature with respect to which observables can be dressed.

CLPW model of observer is somewhat ad hoc; would like to identify more universal features for determining whether an observer is present.

This talk:

- ▶ Argue that nonequilibrium features of background geometry/state lead to effective notion of observer.
- ▶ Describe an explicit cosmological model with an emergent observer.
- ▶ Motivate some work in progress related to half-sided modular inclusion in nonequilibrium settings and their relation to observers.

Explicit observer: crossed product

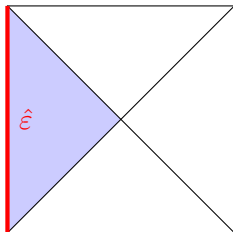
Kinematical algebra:

$$\mathcal{A} = \mathcal{A}_{\text{QFT}} \otimes \mathcal{B}(\mathcal{H})$$

▶ $\mathcal{B}(\mathcal{H}) = \langle \hat{\varepsilon}, \hat{t} \rangle$, with $[\hat{\varepsilon}, \hat{t}] = i$

▶ Still a type III₁ factor

Interpretation: $\hat{\varepsilon}$ the observer
Hamiltonian, \hat{t} the observer clock.



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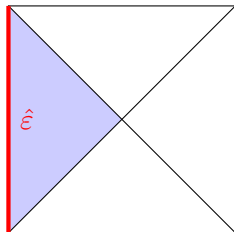
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Constraint is the boost generator $h + \hat{\varepsilon}$, with h the generator for the quantum fields.

Gauge-invariant operators commute with the constraint. Algebra generated by

$$\mathcal{A}_{\text{grav}} = \left\langle e^{i\hat{t}h} a e^{-i\hat{t}h}, \hat{\varepsilon} \right\rangle, \quad a \in \mathcal{A}_{\text{QFT}}$$

Explicit observer: crossed product

Semifinite: algebra has a trace because constraint operator $h + \hat{\varepsilon}$ is the modular Hamiltonian of a weight with density matrix

$$\rho_{\text{tot}} = \rho_{\text{BD}} \otimes e^{-\hat{\varepsilon}}$$

Since operators in $\mathcal{A}_{\text{grav}}$ commute with the density matrix, state is tracial, i.e.

$$\text{Tr}(\rho_{\text{tot}}ab) = \text{Tr}(a\rho_{\text{tot}}b) = \text{Tr}(\rho_{\text{tot}}ba)$$

Leads to conclusion that crossed product algebra is Type II.

More formally: Argue for tracial property using Tomita-Takesaki theory and modular flow. **Centralizer** of the weight associated with ρ_{tot} is semifinite because consists of operators commuting with modular flow.

Explicit observer: crossed product

Nonequilibrium: Weight defined by ρ_{tot} is not normalizable:
Boltzmann density matrix $e^{-\hat{\epsilon}}$ is not trace class:

$$\text{Tr}(e^{-\hat{\epsilon}}) = \int dx \langle x | e^{-\hat{\epsilon}} | x \rangle = \int dx e^{-x} \delta(x - x)$$

Since ρ_{tot} defines the equilibrium state, find that all normalizable states are nonequilibrium.

Nonequilibrium occurs both for CLPW setup, where observer energy is bounded below, or black hole setups where energy is $(-\infty, \infty)$.

Expect effective observers in other nonequilibrium settings.

Will describe an example involving slow-roll inflation and explain how to extract an effective observer by expressing the algebra as a crossed product.

Inflationary algebra

Chen and Penington [2024]: consider scalar field ϕ in dS, suppress back-reaction with $G \rightarrow 0$.

In $4d$, can take the limit holding Λ constant, with slow-roll parameters

$$\varepsilon = \frac{1}{2G} \left(\frac{V_{,\phi}}{V} \right)^2, \quad \eta = \frac{1}{G} \frac{V_{,\phi\phi}}{|V|}$$

going to zero; e.g. $V(\phi) = \frac{\Lambda}{8\pi G} + c\phi$

Admits a description in terms of scalar field on a dS_4 background with linear potential $V = c\phi$, along with free gravitons.

Here, will analyze a simpler model in dS_2 , contains the essential features, but eliminates need to deal with gravitons, higher angular momentum modes

Inflationary algebra

dS_2 metric in conformal coordinates,

$$ds^2 = \frac{\ell^2}{\cos^2(T)} (-dT^2 + d\chi^2)$$

$T \in (-\frac{\pi}{2}, \frac{\pi}{2})$, χ is 2π -periodic

Scalar field action:

$$\begin{aligned} S &= \int d^2x \sqrt{-g} \left(-\frac{1}{2} \nabla_a \phi \nabla^a \phi - c\phi \right) \\ &= \int dT \int_0^{2\pi} d\chi \left(\dot{\phi}^2 - (\phi')^2 - \frac{c\ell^2}{\cos^2(T)} \phi \right) \end{aligned}$$

Theory is free: quantize by imposing canonical commutation relations $[\phi(\chi_1), \dot{\phi}(\chi_2)] = i\delta(\chi_1 - \chi_2)$ on $T = 0$ slice.

Bunch-Davies weight

Also need to specify a vacuum state; natural choice comes from the Bunch-Davies wavefunction computed using Euclidean path integral

$$\Psi_{\text{BD}}[\phi] \sim \exp \left[-\sqrt{2\pi} c \ell^2 \phi_0 - \sum_{n>0} n \phi_n \phi_{-n} \right]$$

- ▶ Wavefunction is $SO(2,1)$ -invariant
- ▶ **Nonequilibrium**: non-normalizable due to zero mode ϕ_0
- ▶ No normalizable states are $SO(2,1)$ invariant
- ▶ Satisfies Hadamard condition at short distances
 $\langle \phi(T, \chi) \phi(0, 0) \rangle \sim \log \left(-(T - i\epsilon)^2 + \chi^2 \right)$

Bunch-Davies weight

For operators in static patch, Bunch-Davies weight satisfies a KMS condition for the time-translation Killing vector

$$\langle \Psi_{\text{BD}} | a(t) b | \Psi_{\text{BD}} \rangle = \langle \Psi_{\text{BD}} | b a(t + 2\pi i) | \Psi_{\text{BD}} \rangle$$

Hence, modular Hamiltonian for this weight is just the time-translation generator, $h = 2\pi H_\xi$

Operators commuting with H_ξ are the centralizer of the BD weight, and comprise the gauge-invariant algebra $\mathcal{A}_{\text{grav}}$. As before $\mathcal{A}_{\text{grav}}$ will have a trace since consists of operators commuting with modular flow.

Construction of the centralizer

Various ways to see that $\mathcal{A}_{\text{grav}}$ is nontrivial in this context.

Most prosaic: perturbative construction as power series in $\frac{1}{c\ell^2}$

Boost generator takes the form

$$\begin{aligned}H_\xi &= H_0 + c\ell^2 (\hat{x} - \hat{x}') \\H_0 &= \int_{-\pi}^{\pi} d\chi \cos \chi \left(\frac{1}{2} \pi(\chi)^2 + \frac{1}{2} \phi'(\chi)^2 \right) \\ \hat{x} &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\chi \cos \chi \phi(\chi)\end{aligned}$$

Construction of the centralizer

Recursion relation to solve for \hat{a} : write

$$\hat{a} = \sum_{n=0}^{\infty} \frac{1}{(c\ell^2)^n} \mathbf{a}_n$$

and impose

$$[\hat{x}, \mathbf{a}_n] = \begin{cases} 0 & n = 0 \\ -[H_0, \mathbf{a}_{n-1}] & n \geq 1 \end{cases}$$

Can solve this relation with each \mathbf{a}_n a product of $n + 1$ smeared field operators:

$$\hat{a} = \int d\chi f \phi + \frac{1}{c\ell^2} \int d\chi_1 \int d\chi_2 (g\pi)_1 (g\pi)_2 + \frac{1}{(c\ell^2)^2} \iiint \dots$$

Note: Each \mathbf{a}_n is more nonlocal than the previous term \mathbf{a}_{n-1} , and also suppressed by a power of $\frac{1}{c\ell^2}$. Centralizer becomes more local when potential slope is large \rightarrow steeper slope leads to a more accurate clock

Integrable weights

Can also argue for existence of nontrivial $\mathcal{A}_{\text{grav}}$ using modular time integral.

Linear potential causes scalar field to decay at early and late times; marginalization over time

$$\mathcal{T}(\mathbf{a}) = \int_{-\infty}^{\infty} dt e^{iHt} \mathbf{a} e^{-iHt}$$

converges on a dense set of operators, e.g. consider $e^{-(\int f\phi)^2}$

Contrast with a standard massive scalar: static patch equilibrates, causing the integral to diverge.

Weights where modular time integral converges are known as **integral weights**, and image is $\mathcal{A}_{\text{grav}}$ which is type II.

Possesses a trace τ defined via the relation

$$\omega_{\text{BD}} = \tau \circ \mathcal{T}$$

Intrinsic observer

Use the shift symmetry $\theta_s(\phi(x)) = \phi(x) + s$ to identify a class of observer Hamiltonians: defined as eigenoperators of θ_s in $\mathcal{A}_{\text{grav}}$. This definition is in analogy to the crossed product, where θ_s is generated by the time operator \hat{t} .

Find these as follows:

- ▶ Choose a weight ω_0 on shift-symmetric algebra $\mathcal{A}_{\text{grav}}^\theta$
- ▶ Compose with $\mathcal{S} = \int ds \theta_s(\cdot)$ to get a weight on $\mathcal{A}_{\text{grav}}$:
 $\omega = \omega_0 \circ \mathcal{S}$
- ▶ Define a density matrix via $\omega(a) = \tau(\rho_\omega a)$
- ▶ ω is invariant under θ_s , while $\tau(\theta_s(\cdot)) = e^{-s} \tau(\cdot)$. Use this to show that ρ_ω is an eigenoperator:

$$\theta_s(\rho_\omega) = e^{-s} \rho_\omega$$

- ▶ Identify the observer Hamiltonian as $\hat{\varepsilon} = \log \rho_\omega$

Crossed-product description

Unitary operators $v_t = e^{it\hat{\varepsilon}}$ generate modular flow of ω_0 on fixed point algebra $\mathcal{A}_{\text{grav}}^\theta$, eigenoperators for θ_s .

Landstad's theorem $\implies \mathcal{A}_{\text{grav}}$ is a modular crossed product

$$\mathcal{A}_{\text{grav}} = \mathcal{A}_{\text{grav}}^\theta \rtimes \mathbb{R}$$

Crossed-product description

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Frame dependence:

Choice of observer depends on weight ω_0 on $\mathcal{A}_{\text{grav}}^\theta$. Can shift $\hat{\varepsilon} \rightarrow \hat{\varepsilon} + \hat{\gamma}$ with $\hat{\gamma} \in \mathcal{A}_{\text{grav}}^\theta$, new Hamiltonian still a θ_s -eigenoperator

Corresponds to a different choice of weight on $\mathcal{A}_{\text{grav}}^\theta$; $\hat{\gamma}$ determines the **Connes cocycle** between the two weights. Related to the background independence of crossed product described by Witten [Witten 2021]; should instead interpret this as **frame independence**.

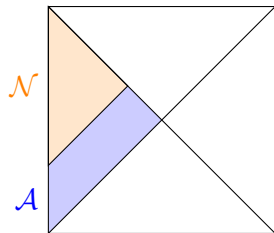
Representing the algebra as a crossed product can be viewed as a decomposition of the algebra into observer and quantum fields.

A preferred frame?

Possible that a preferred $\hat{\varepsilon}$ can be found using **half-sided modular inclusions**:

$\mathcal{N} \subset \mathcal{A}$ and modular flow of \mathcal{N} satisfies

$$\Delta_{\mathcal{A}}^{-is} \mathcal{N} \Delta_{\mathcal{A}}^{is} \subset \mathcal{N} \text{ for } s \geq 0$$



ANE operator on the future horizon is

$$2\pi P = \log \Delta_{\mathcal{N}} - \log \Delta_{\mathcal{A}} \geq 0$$

Comparing to crossed product, would find that P -translation-invariant algebra \mathcal{A}^T generated by $\langle \hat{\varepsilon}, \hat{t} \rangle$, and

$$\mathcal{A}^T \wedge \mathcal{A}^\varphi = \langle \hat{\varepsilon} \rangle,$$

singling out a preferred observer operator $\hat{\varepsilon}$.

Nonequilibrium HSMI

Motivates closer consideration of HSMI in nonequilibrium settings.

HSMI always involves a vacuum state satisfying $P|\Omega\rangle = 0$.

In nonequilibrium setting, $|\Omega\rangle$ is not normalizable, instead viewed as a semifinite weight (analogous to momentum eigenstates).

Such weights should arise in many situations of interest:

- ▶ Slow-roll inflation example from this talk
- ▶ Unruh state for evaporating black hole
- ▶ Asymptotically flat Kerr black hole: superradiance
- ▶ Schwarzschild-de Sitter: different temperature horizons

Work in progress establishing properties. Main questions:

- ▶ Algebraic type: conjectured to be type III₁.
- ▶ Algebra at infinity: is $\mathcal{A}_\infty = \mathcal{A}^\tau$? Appears true at least for a preferred class of vacua.

Summary and outlook

Summary

- ▶ Discussed connection between observers in semiclassical gravitational algebras and nonequilibrium features of the background
- ▶ Gave an explicit example involving slow-roll inflation in which observer is an emergent degree of freedom
- ▶ Commented on connection between observer and properties of HSMI

Future work:

- ▶ Complete characterization of nonequilibrium HSMI, and establish their existence in setting mentioned above (Unruh state, Kerr, SdS, ...)
- ▶ Equilibrium observers? Possible to have centralizers in normalizable states. Connection to present discussion?
- ▶ Connect to path integral and observer rules and problems with closed universes