a de Sitter anti-scrambling algebra

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IAS

Based on W.I.P. with Wentao Cui

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Introduction and Summary

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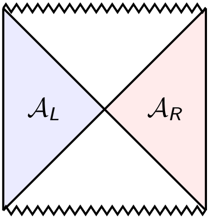

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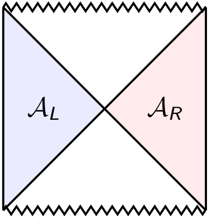

- The algebras factorize: $\mathcal{A}_L' = \mathcal{A}_R$, $\mathcal{A}_R' = \mathcal{A}_L$, and $\mathcal{A}_L \cap \mathcal{A}_R = \mathbb{C}$ (compare to $\mathcal{H} = \mathcal{H}_L \otimes \mathcal{H}_R$).

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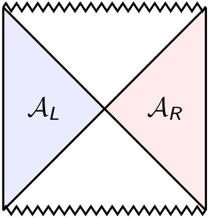

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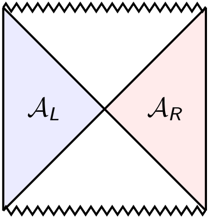

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- The algebra $\mathcal{A}_R$ is a large-N limit of single-trace operators in the R CFT in the TFD state. [Leutheusser-Liu '21]
- A subalgebra gives a precise notion of a subsystem (or horizon) in quantum gravity, even when quantum metric fluctuations and backreaction are taken into account.

- When G_N corrections are included, the algebras $\mathcal{A}_R$ and $\mathcal{A}_L$ still factorize. [Penington-Tabor '25, DK '23, Penington-Witten '23, '24]

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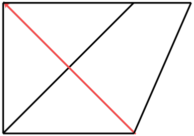

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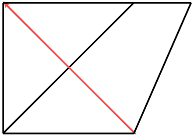
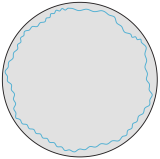


- In JT gravity, the metric is exactly quantized, and wormhole corrections may be considered.



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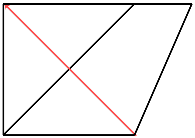

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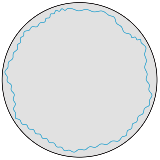
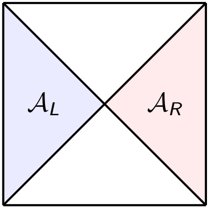


- By studying algebras in AdS/CFT, we can motivate the duality from the bottom-up.

- An initial study of algebras in dS was undertaken in [Chandrasekaran-Longo-Penington-Witten ‘22].



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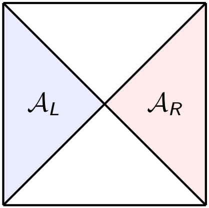

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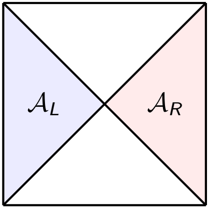

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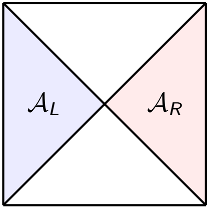

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- I will study G_N corrections to this picture, following the methods of [Penington-Tabor '25]. We work in a strict $G_N \to 0$ limit, holding $x := G_N e^{\frac{2\pi T}{\beta_c}}$ fixed. Operators can have time separations of order T. We can work exactly in x, or order by order in x.

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- In QFT in dS, the Hartle-Hawking vacuum state is thermal (or KMS) with respect to time translations along the observer's worldline. This is a consequence of the temperature of the geometric horizon. Working exactly in x, the time-translation symmetry persists, but the KMS condition fails. Order by in order in x, KMS holds.

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- What does this imply for observer-centric dS holography?

[Anninos-Hartnoll-Hofman '11] [Witten '23] [Narovlansky-Verlinde '23] [Harlow-Usatyuk-Zhao '25] [Abdalla-Antonini-Iliesiu-Levine '25] [Akers-Bueller-DeWolfe-Higginbotham-Reinking-Rodriguez '25] [Narovlansky '25] [Engelhardt-Gesteau-Harlow '25] [Harlow '26] [Zhao '26] [Goto-Milekhin-Verlinde-Xu '26]

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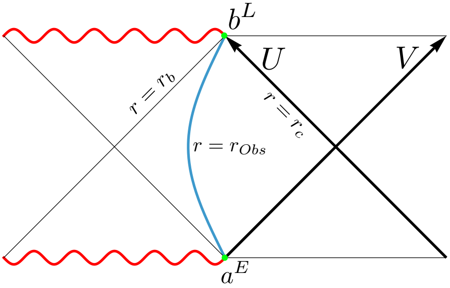

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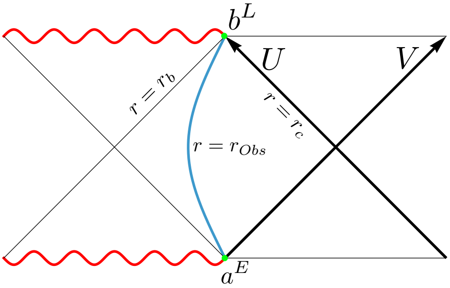

- The observer's algebra $\mathcal{A}_0$ is generated by local QFT operators between $r_{\rm Obs}$ and r_c. To compute correlation functions, Wick rotate the time t to $-i\tau$, and compactify $\tau \sim \tau + \beta_c$. The Hilbert space $\mathcal{H}_0$ can be recovered from the GNS construction.

An observer in de Sitter space

- We are interested in correlation functions of operators with large time separations. Given $\mathcal{O} \in \mathcal{A}_0$, define

$$\mathcal{O}^E := e^{-iHT/2}\,\mathcal{O}\,e^{+iHT/2},$$
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$$\lim_{T\to\infty} \langle \cdots \mathcal{O}_4^E \mathcal{O}_3^L \mathcal{O}_2^E \mathcal{O}_1^L \rangle_{\mathrm{QFT}} = \langle \cdots \mathcal{O}_4^E \mathcal{O}_2^E \rangle_{\mathrm{QFT}} \, \langle \cdots \mathcal{O}_3^L \mathcal{O}_1^L \rangle_{\mathrm{QFT}} \,.$$

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- We will use a doubled Hilbert space

$$\mathcal{H} := \mathcal{H}_A \otimes \mathcal{H}_B,$$

  where $\mathcal{A}_{0,A}$ acts on $\mathcal{H}_A$ and $\mathcal{A}_{0,B}$ acts on $\mathcal{H}_B$.

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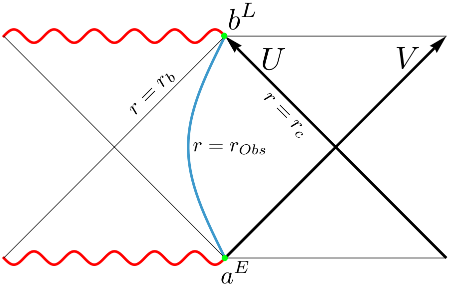

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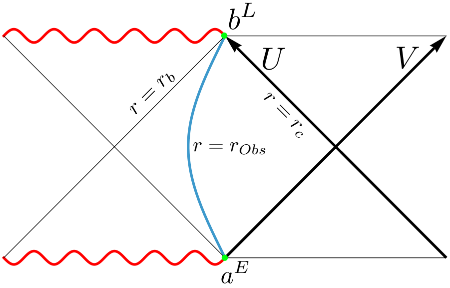

- Focus on the early operators. We make an ansatz:

$$T_{UU} = -\frac{1}{r_c^2}\, P_U(\Omega)\, \delta(U),$$

where $P_U(\Omega)$, for every $\Omega \in S^2$, is an ANEC operator,

$$P_U(\Omega) := -r_c^2 \int_{-\infty}^{\infty} dU\, T_{UU}(U, V = 0, \Omega) \;\sim\; O\Big(e^{\pi T/\beta_c}\Big)\,.$$

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- T_{UU} sources a perturbation in linearized gravity with positive c.c. ($\Lambda = \frac{3}{\ell_{dS}^2}$)

$$ds^2 = ds_{SdS}^2 + \delta(U)\, f_U(\Omega)\, dU^2,$$

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- We send $G_N \to 0$ with $G_N\, e^{2\pi T/\beta_c}$ fixed.
- The SdS geometry comes in a one-parameter family. r_c obeys

$$\frac{1}{\sqrt{\Lambda}} < r_c < \sqrt{\frac{3}{\Lambda}},$$

and $\left(-\nabla^2_{S^2} + 1 - r_c^2\Lambda\right)$ is invertible strictly within this range.

An observer in de Sitter space

- With both early- and late-time shocks, the metric is

$$ds^2 = ds^2_{SdS} + \delta(U)\, f_U(\Omega)\, dU^2 + \delta(V)\, f_V(\Omega)\, dV^2,$$

and the action is

$$I = 16\pi G \int d\Omega\ P_V(\Omega)\, \frac{1}{-\nabla^2_{S^2} + 1 - \Lambda r_c^2}\, P_U(\Omega)\ \sim\ O(1).$$

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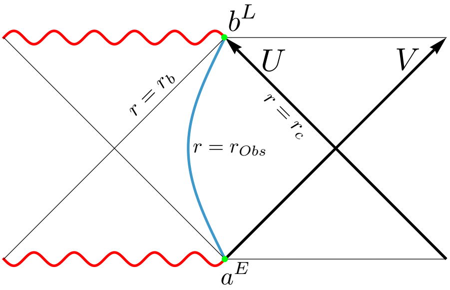

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- The definitions of a^E and b^L involve conjugations by $e^{iH_A T/2}$ or $e^{-iH_B T/2}$. We can absorb these operators into the S-matrix:

$$S := e^{iH_A T/2}\, e^{-iH_B T/2}\, \mathcal{S}\, e^{-iH_A T/2}\, e^{iH_B T/2}.$$

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$$S = e^{i\delta}, \qquad \delta = 16\pi G\, e^{2\pi T/\beta_c} \int d\Omega\ P_V(\Omega)\, \frac{1}{-\nabla^2_{S^2} + 1 - \Lambda r_c^2}\, P_U(\Omega).$$

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$$\langle \cdots a_1^E\, b_2^L a_3^E\, b_4^L \rangle_{\mathrm{QG}} = \langle \cdots S\, a_1\, S^{-1}\, b_2\, S\, a_3\, S^{-1}\, b_4 \rangle_{\mathrm{QFT}}\,.$$

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- Given $a \in \mathcal{A}_{0,A}$ and $b \in \mathcal{A}_{0,B}$, define

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Then we may simply write

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- The observer's algebra $\mathcal{A}$ is generated by $\tilde{a}$ and $\tilde{b}$ for all $a \in \mathcal{A}_{0,A}$ and all $b \in \mathcal{A}_{0,B}$.

An observer in de Sitter space

- Recall that

$$\frac{1}{\sqrt{\Lambda}} < r_c < \sqrt{\frac{3}{\Lambda}}.$$

- Consider the limit $r_c \to \sqrt{\frac{3}{\Lambda}}$ with $G_N \to 0$ and M fixed. The SdS geometry becomes global dS. The observer sits at $r = 0$. The eikonal phase becomes

$$\delta = \frac{2\ell_{dS}}{M} e^{2\pi T/\beta_c} \int d\Omega_1 d\Omega_2 \, P_U(\Omega_1) P_V(\Omega_2) \cos\Theta(\Omega_1, \Omega_2).$$

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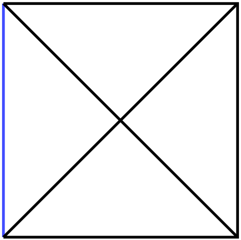

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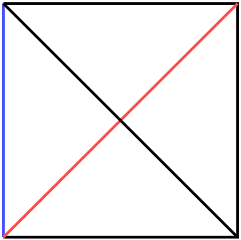

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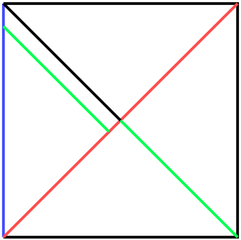

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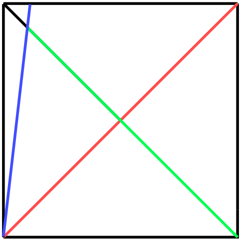

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- The s-wave sector decouples.

$$\delta = -\frac{4\pi}{\Delta S_c} \frac{e^{2\pi T/\beta_c}}{\Lambda} P_U P_V,$$

with

$$P_U := \int d\Omega \, P_U(\Omega), \quad P_V := \int d\Omega \, P_V(\Omega)$$

A toy model of the s-wave sector

- As a toy model of the s-wave sector, we will take QFT A to be a right-moving 2D chiral CFT. QFT B will be a left-moving 2D chiral CFT. The S matrix is

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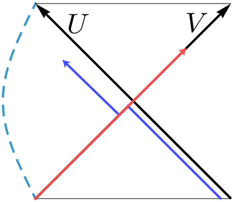

dS_2, $x < 0$
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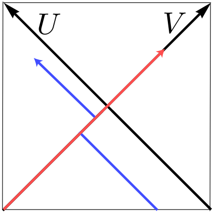

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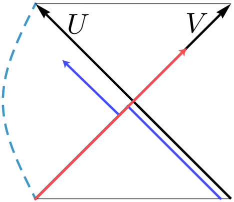

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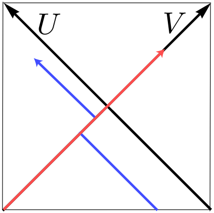

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We will study this model both exactly in x, and perturbatively in x.

Review of 2D chiral CFT

- Fields are functions of the affine coordinate on the horizon, U. A primary field $\phi_j(U)$ with weight $h_j \in \mathbb{Z}_{\geq 0}$ transforms as

$$e^{iyP}\phi_j(U)e^{-iyP} = \phi_j(U-y)$$

$$e^{iyB}\phi_j(U)e^{-iyB} = e^{-h_j y}\phi_j(e^{-y}U)$$

$$e^{iyK}\phi_j(U)e^{-iyK} = \left(\frac{1}{1-Uy}\right)^{2h_j}\phi(\frac{U}{1-Uy})$$

The Hermitian generators P, B, and K obey

$$[B,P] = i\,P,$$
$$[B,K] = -\,i\,K,$$
$$[P,K] = 2i\,B.$$

The primaries are Hermitian w.l.o.g.
I will suppress the subscripts on the fields when it will not cause confusion.

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$$\langle \phi_{j_1}(U_1)\phi_{j_2}(U_2)\rangle = \delta_{j_1 j_2}\left(-\frac{1}{(U_1 - U_2 - i\epsilon)^2}\right)^{h_{j_1}}$$
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- The Hilbert space is generated by the vacuum, $|\Omega\rangle$, and $\phi_i(U)\,|\Omega\rangle$ for U in any interval in $\mathbb{R}$ (Reeh–Schlieder theorem and state-operator correspondence).

The commutant of the observer's algebra

- For $x > 0$, the operators $\tilde{a}$ and $\tilde{b}$ for $a \in \mathcal{A}_{0,A}$ and $b \in \mathcal{A}_{0,B}$ can be interpreted as operators dual to insertions on an AdS boundary.

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- For $x < 0$ (dS), we may say that $\widetilde{a'}$ and $\widetilde{b'}$ generate $\mathcal{A}'$ *order by order in perturbation theory.* There is a pair of factorizing algebras.

The KMS condition in 2D Chiral CFT

- Consider a thermal two-point function in finite-dimensional quantum mechanics:

$$F(t_1, t_2) = \text{Tr } e^{-2\pi H}\mathcal{O}_1(t_1)\mathcal{O}_2(t_2),$$

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- Note that

$$\langle \phi_1(U_1) \cdots \phi_n(U_n) \rangle = \langle \phi_1(0) e^{iU_1 P} \cdots \phi_n(U_n) \rangle$$

is holomorphic for U_1 in the lower-half-plane, because $P \leq 0$. Thus, we may continue $U_1 \to U_1 e^{-\pi i} = -U_1$.

$$\langle \phi_1(U_1) \cdots \phi_n(U_n) \rangle \to \langle \phi_1(e^{-i\pi} U_1) \cdots \phi_n(U_n) \rangle = \langle \cdots \phi_n(U_n) \phi_1(e^{-i\pi} U_1) \rangle$$

Hence, we may analytically continue $U_1 \to U_1 e^{-2\pi i}$, and the KMS condition is established.

The KMS condition and the OTOC

- Consider the OTOC

$$
\begin{aligned}
&\langle \tilde{\phi}^A(U_1)\tilde{\phi}^B(V_2)\tilde{\phi}^A(U_3)\tilde{\phi}^B(V_4)\rangle \\
&= \langle \phi^A(U_1)\phi^B(V_2)e^{ixP_AP_B}\phi^A(U_3)\phi^B(V_4)\rangle \\
&= \langle \phi^A(U_1)\phi^B(V_2)\phi^A(U_3 - xP_B)\phi^B(V_4)\rangle \,,
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with $U_1, U_3 > 0$ and $V_2, V_4 < 0$.

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- If $x > 0$, or if we work to finite order in x, the commutator vanishes. Continue $U_1 \to e^{-i\pi}U_1$ to obtain

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- The KMS condition involves analytic continuation: $t \to t - \beta i$. Have we analytically continued into a "fake" region? [Lin-Stanford '23].

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- **Example.** An entangled state in two copies of a finite-dimensional Hilbert space whose reduced density matrix on either copy is full-rank.

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$$a\,|\Psi\rangle = 0 \implies a = 0, \quad \forall a \in \mathcal{A}$$
- **Theorem.** $|\Psi\rangle$ is cyclic for $\mathcal{A}$ iff $|\Psi\rangle$ is separating for $\mathcal{A}'$, the commutant of $\mathcal{A}$.
- **Example.** An entangled state in two copies of a finite-dimensional Hilbert space whose reduced density matrix on either copy is full-rank.
- **Theorem.** (Reeh-Schlieder): In chiral CFT, the vacuum $|\Omega\rangle$ is cyclic and separating for the algebra of operators of an interval on the horizon, $U \in [U_1, U_2]$.

Tomita-Takesaki theory in chiral CFT

- Let $|\Psi\rangle$ be cyclic and separating for $\mathcal{A}$. The *Tomita operator* S_Ψ is defined by

$$S_\Psi a |\Psi\rangle = a^\dagger |\Psi\rangle .$$

- The *modular operator* is defined by $\Delta_\Psi := S_\Psi^\dagger S_\Psi$.
- Matrix elements of Δ_Ψ are computed using

$$\langle\Psi|a_1 \Delta_\Psi a_2|\Psi\rangle = \langle\Psi|a_2 a_1|\Psi\rangle , \quad a_1, a_2 \in \mathcal{A}.$$

- For a chiral CFT, let $\mathcal{A}$ be the algebra of operators associated to $U > 0$. Then $\Delta_\Omega = e^{2\pi B}$. Time translation is given by $a(t) = e^{-itB} a e^{itB}$. Thus,

$$\langle\Psi|a_1(-2\pi i) a_2|\Psi\rangle = \langle\Psi|a_2 a_1|\Psi\rangle ,$$

which is the KMS condition.

Tomita-Takesaki theory in the observer's algebra

- Let $\mathcal{A}$ be the observer's algebra, with $x < 0$. The vacuum $|\Omega\rangle = |\Omega_A\rangle \otimes |\Omega_B\rangle$ is cyclic for $\mathcal{A}$. Let $a \in \mathcal{A}_{0,A}$ and $b \in \mathcal{A}_{0,B}$.

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- **Conjecture.** $\mathcal{A}' = \mathbb{C}$.

Summary and Conclusion

- A static observer at the North Pole in dS has a horizon, and an associated static patch algebra $\mathcal{A}$. The commutant $\mathcal{A}'$ represents the other static patch. $\mathcal{A}$ and $\mathcal{A}'$ are a pair of factorizing algebras, and vacuum correlators obey the KMS condition.

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