# Quantum chaos and wormholes

Zhenbin Yang

Observers, wormholes and complex saddles in cosmology
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# Motivation

- Quantum chaos asks how generic many-body systems thermalize and develop random-matrix spectral correlations.
- Black holes turn these diagnostics into geometric statements about horizons, scatterings and wormholes.
- We will discuss this connection in four examples, and then the puzzles.

# Roadmap

1. Setup: the TFD, and boost evolution.
2. Two-point decay: QNMs
3. OTOCs: shockwaves and the scrambling time.
4. Spectral statistics: the double cone.
5. ETH statistics: the Stanford wormhole.
6. Puzzles: factorization and interior observers.

# Euclidean Black Hole

The canonical partition function is a Euclidean path integral on a thermal circle:

$$Z(\beta) = \mathrm{Tr}\, e^{-\beta H}.$$

In holography, the gravitational saddle fills in the thermal circle:

$$S^1_\beta \times S^{d-1} \longrightarrow \text{Euclidean black hole.}$$

The thermal circle is contractible in the bulk; cutting it prepares a state in the doubled Hilbert space.

Refs: Gibbons–Hawking; Maldacena (2001)

# Cutting Prepares the TFD

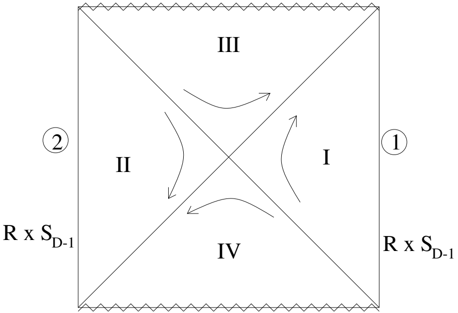
Cutting the Euclidean thermal circle in half prepares

$$|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\beta E_n/2} |n\rangle_L |n\rangle_R .$$

Analytic continuation gives the Lorentzian eternal black hole:

$$\theta = \frac{2\pi}{\beta}\tau \longrightarrow \theta = i\frac{2\pi}{\beta}t.$$

This is a state in a double system $\mathcal{H} \otimes \mathcal{H}^*$, sometimes called the Hartle-Hawking vacuum.




The two exterior regions are dual to two noninteracting CFTs in the entangled TFD state.

Refs: Maldacena, “Eternal black holes in anti-de Sitter” (2001), Fig. 1

# Thermalization of Two-Point Functions

Consider

$$\langle W(t)W(0)\rangle.$$

- In a mixing classical system, late-time decay is controlled by Ruelle–Pollicott resonances.
- In holographic thermal states, black-hole quasi-normal modes appear as poles of retarded boundary Green's functions.
- In doubled language, $U \otimes U^*$ is the quantum Liouvillian; in the eternal black hole it becomes the boost evolution, the quasi-normal modes become a generalized eigenvalue of the Liouvillian operator.

# A Simple Thermal Correlator

For an operator of dimension $\Delta$ in the simplest near-AdS$_2$ setting,

$$\langle W(t)W(0)\rangle_\beta \sim \frac{1}{\left(i \sinh \frac{\pi t}{\beta}\right)^{2\Delta}}.$$

At late times,

$$\langle W(t)W(0)\rangle_\beta \sim e^{-\frac{2\pi\Delta}{\beta}t}\left(1+\sum_{n\geq 1} c_n e^{-\frac{2\pi n}{\beta}t}\right).$$

More general black holes replace this explicit series by the QNM spectrum.
In the heavy-operator limit, two-sided correlators are controlled by geodesic lengths through the wormhole:

$$\langle W_L(T)W_R(0)\rangle \sim e^{-m\,\ell_{\rm geo}(T)} \sim e^{-mT}.$$

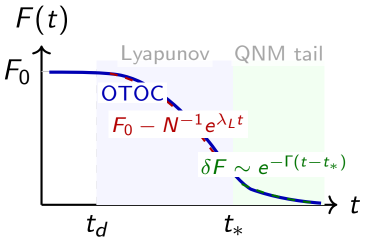
# OTOC and Quantum Lyapunov Growth

The TFD state is unstable with respect to early and late perturbations. A standard diagnostic is

$$F(t) = \frac{1}{Z}\mathrm{Tr}\left[e^{-\beta H} V(0) W(t) V(0) W(t)\right].$$

At large $N$,

$$F(t) \approx F_0 - \frac{1}{N} e^{\lambda_L t} + \cdots , \qquad t_d \ll t \ll t_* .$$




The scrambling time is when the small perturbation becomes order one.

$$t_* \sim \lambda_L^{-1} \log S \qquad (\lambda_L = 2\pi/\beta \text{ for Einstein gravity}).$$

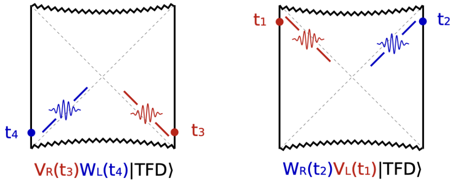
Refs: Shenker–Stanford (2015), Fig. 2; Maldacena–Shenker–Stanford (2015)

# OTOC as an In-Out Overlap

The OTOC is an overlap between two differently ordered states:

$$F(t) = \langle W(t)V(0)\mathrm{TFD} \,|\, V(0)W(t)\mathrm{TFD} \rangle.$$

- In the bulk, the two states are in- and out-states scattering near the bifurcate horizon.
- The in- and out-states becomes shockwaves that generates null shifts of the two horizons, that can thought of as retarded and advanced scrambling modes.




Refs: Shenker–Stanford, "Black holes and the butterfly effect" (2013); Gu–Kitaev–Zhang (2022)

# Dray–'t Hooft Shockwave

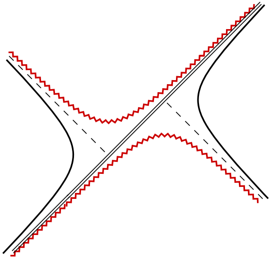
The two particles separated by boundary time $T$ are exponentially boosted near the horizon:

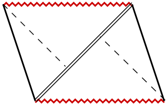
$$s_{\rm cm} \propto e^{\frac{2\pi}{\beta} T}.$$

This leads to a shockwave shift at the null horizons:

$$\Delta x^{\pm} \propto G_N e^{\frac{2\pi}{\beta} T}.$$

The shift becomes order one at the scrambling time.





Refs: Shenker–Stanford (2013), shockwave geometry; Maldacena–Shenker–Stanford (2015)

# Random Matrix Universality

The BGS conjecture states that classically chaotic quantum systems have local spectral fluctuations described by the random-matrix ensemble within the same symmetry class.

$$\Delta E < E_{\mathrm{Th}}.$$

The GUE connected spectral correlator has the universal large-separation tail called spectral rigidity:

$$\langle \rho(E)\rho(E')\rangle_c = -\frac{\sin^2\!\big(\pi(E-E')\rho(E)\big)}{\pi^2(E-E')^2} \sim -\frac{1}{\pi^2(E-E')^2} + \cdots .$$

Refs: Bohigas–Giannoni–Schmit (1984); Mehta, *Random Matrices*

# Spectral Form Factor

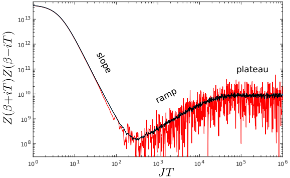
Define

$$g(T) = \langle Z(iT)Z(-iT) \rangle , \qquad Z(iT) = \mathrm{Tr}\, e^{-iHT} .$$

In doubled language,

$$g(T) = \mathrm{Tr}\big(U(T) \otimes U^*(T)\big).$$

- The spectral rigidity Fourier transforms to the ramp.
- In gravity this trace suggests quotienting the two-sided black hole by boost time.



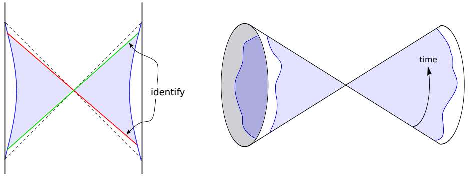

Refs: Cotler et al. (2017); Saad–Shenker–Stanford (2018)

# The Double Cone Saddle

Saad–Shenker–Stanford identified a two-replica semiclassical saddle responsible for the ramp.

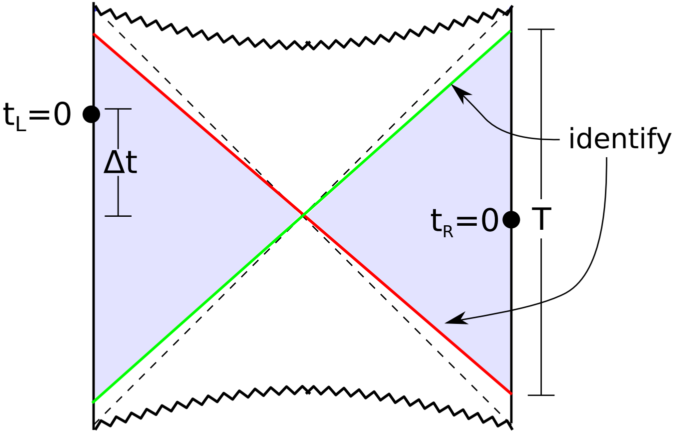
- Start from the two-sided black-hole geometry.
- Quotient by a Killing time translation of period $T$.
- The horizon fixed point is smoothed by a small complex deformation.

The first approximation is a bulk trace of the ordinary boost evolution, $\mathrm{Tr}_{\mathsf{bulk}}\, e^{-iKT}$.




Refs: Saad–Shenker–Stanford (2018), Fig. 6

# Double Cone Picture




The saddle has vanishing classical action; the ramp comes from integrating over a compact zero mode.

Refs: Saad–Shenker–Stanford (2018), Fig. 8

# Origin of the Ramp

The double cone has a zero mode: a relative time shift, or twist, between the two boundaries.

$$|\mathrm{TFD}(\tau)\rangle = e^{-iH_L\tau}\,|\mathrm{TFD}\rangle = e^{-iH_R\tau}\,|\mathrm{TFD}\rangle$$

The quotient by the external time $T$ makes the twist orbit compact:

$$\int_0^T \frac{\mathrm{d}\tau\,\mathrm{d}E}{2\pi} \sim \frac{T\,\Delta E}{2\pi}.$$

The volume of this compact zero mode is the semiclassical origin of the linear ramp.

Refs: Saad–Shenker–Stanford (2018); Stanford–Witten (2019)

# Modified Boost and QNMs

With bulk matter, the complexified double-cone prescription replaces the boost trace by

$$\mathrm{Tr}_{\mathsf{bulk}}\, e^{-i\widetilde{K}T}.$$

- $\widetilde{K} = K - i\epsilon G$ is a non-Hermitian operator due to the contour deformation. Here $G$ generates the local Minkowski time evolution at the Killing horizon.
- For general black holes, its spectrum is the quasi-normal-mode spectrum:
$$\widetilde{K}\,|\psi_n\rangle = \omega_n\,|\psi_n\rangle\,, \qquad \omega_n = \Omega_n - i\Gamma_n.$$

Thus the same QNMs that controlled two-point decay also control one-loop physics around the double cone.

The slowest decaying non-zero mode controls the approach to the universal ramp and is related to the Thouless scale,

$$t_{\mathrm{Th}} \sim \frac{1}{\min_n \Gamma_n}.$$

Refs: Chen–Ivo–Maldacena, JHEP 04 (2024) 124

# From Eigenvalues to Eigenvectors

The double cone explains correlations among energy eigenvalues. But quantum chaos also predicts randomness in energy eigenvectors:

$$O_{mn} = f(\bar{E})\delta_{mn} + e^{-S(\bar{E})/2} g(\bar{E},\omega) R_{mn},$$
$$\mathrm{Tr}\, O(t)O(0) \sim \int \mathrm{d}E\, \mathrm{d}E'\, e^{S(E)+S(E')-S(\frac{E+E'}{2})} \left| g\left(\frac{E+E'}{2}, E-E'\right)\right|^2 .$$

ETH treats $R_{mn}$ as erratic order-one noise:

$$\overline{R_{mn}} = 0, \qquad \overline{|R_{mn}|^2} = 1, \qquad R_{mn} = R^*_{nm} \quad (O = O^\dagger).$$

To see the statistics of $R_{mn}$, we consider

$$\overline{|\mathrm{Tr}\, V(t)W(0)|^2}_{\mathrm{conn}} \sim \rho(E)\rho(E')\, |g_V(\bar{E},\omega) g_W(\bar{E},\omega)|^2 .$$

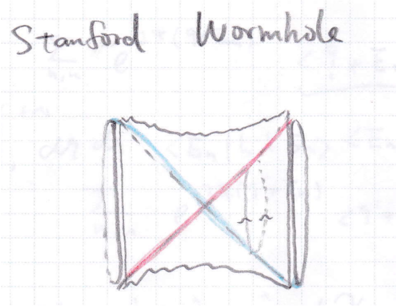
To see this in gravity, we keep the same boost-trace idea but insert operator sources.

# Stanford Wormhole Observable

The wormhole contribution to a squared two-point function can be viewed as

$$\mathrm{Tr}_{\text{bulk}}\left(e^{iKT}V_L V_R e^{-iKT}W_L W_R\right).$$

- $K$ is the boost generator of the two-sided black hole.
- $V_L V_R, W_L W_R$ creates null momenta generating small horizon shifts.
- Compared with the double cone, the trace now contains operator-source kicks.




Refs: Stanford, arXiv:2008.08570; Stanford-Yang unpublished

# Scrambling-Time Threshold

The pair of operators $V_L V_R, W_L W_R$ prevents the two boost evolutions from cancelling at late time.

- Each operator source must first grow into an order-one shockwave after a scrambling time $t_*$.
- The effective propagation time is $T - 2t_*$, after which the bulk particle displays quasi-normal-mode decay.
- A stable long wormhole appears only after the bulk particle loops decay:

$$T \gtrsim 2t_* + t_{\mathrm{Th}}.$$

This is a bulk prediction of the time scale when ETH ansatz becomes valid.

Refs: Stanford, arXiv:2008.08570; Stanford-Yang unpublished

# Puzzle 1: Factorization

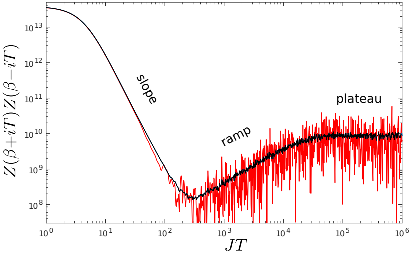
Wormholes give connected contributions to products of observables:

$$\mathrm{grav}[Z_1 Z_2]_{\mathrm{conn}} \neq 0.$$

This is natural in an ensemble average,

$$\overline{Z_1 Z_2} \neq \overline{Z_1}\,\overline{Z_2},$$

but puzzling for a single microscopic quantum system.




The question is whether the gravitational path integral computes an ensemble, a coarse-grained answer, or a more subtle single-theory observable.

Refs: Maldacena–Maoz; Witten–Yau; Saad–Shenker–Stanford; Liu; Kudler-Flam–Witten...

# Puzzle 2: Interior Observers

Wormholes also lead to puzzle about interior observers.

- Firewall/time ordering ambiguity
  In QFT, when we analytic continue euclidean correlator to lorentzian correlator, we encounter branch cuts, that allows us to define time ordering.
  In QG, especially on a wormhole, there could exist multiple branch cuts that can fluctuate. This makes the bulk time ordering ambiguous.
- Closed universe observers
  It's still tempting to use boundary Hilbert space to describe bulk observers. But if the observer lives in a closed universe, this becomes more confusing.

# Conclusion

| Quantum chaos | Gravity |
|---|---|
| Ruelle–Pollicott resonances | Quasi-normal modes |
| OTOC growth | Shockwave and scrambling |
| Spectral form factor ramp | Double cone |
| ETH matrix-element noise | Stanford wormhole |

Thank you