

# Old-fashioned Observer-centric Quantum Cosmology

Lausanne

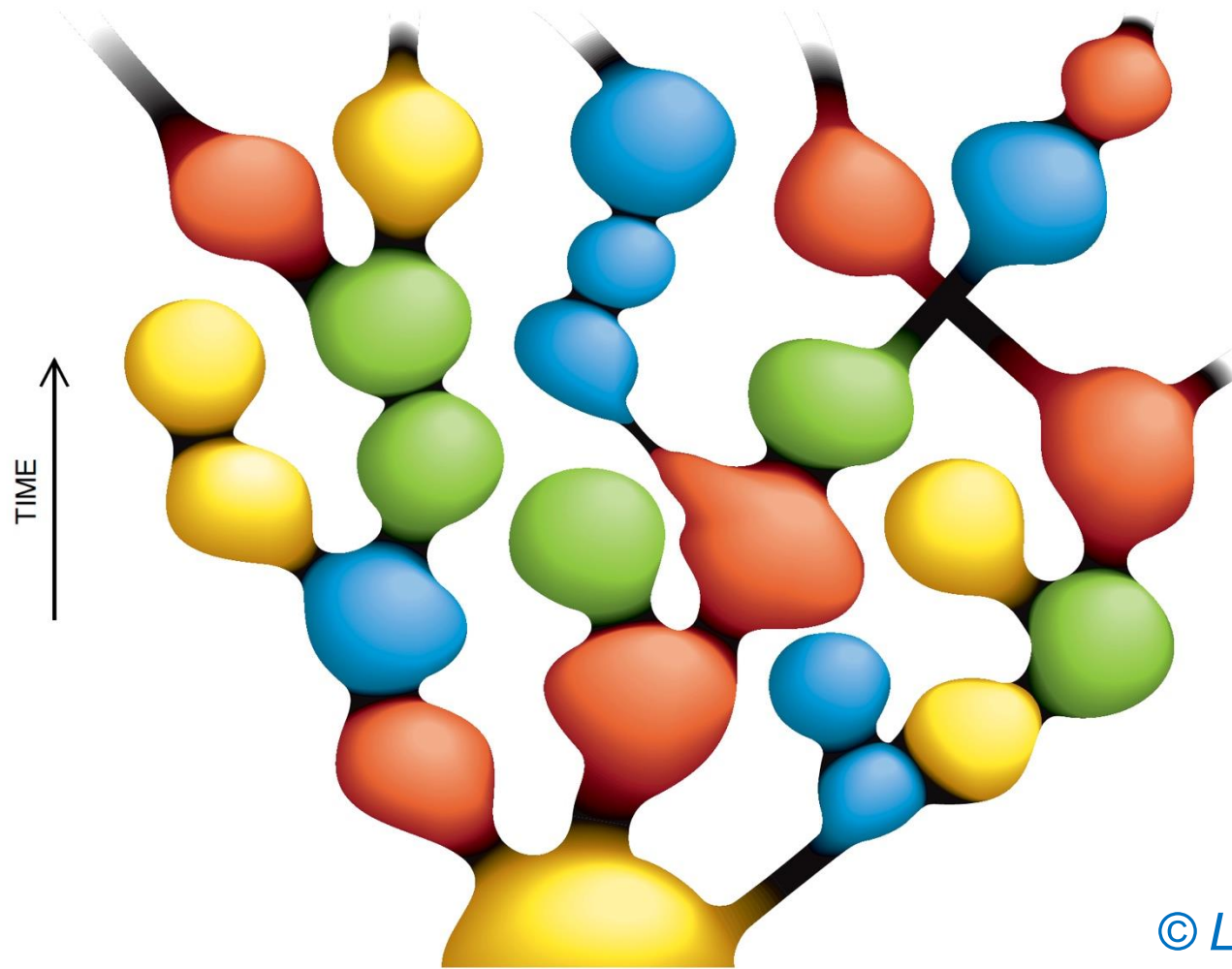
May 2026

Thomas Hertog  
Institute for Theoretical Physics  
KU Leuven

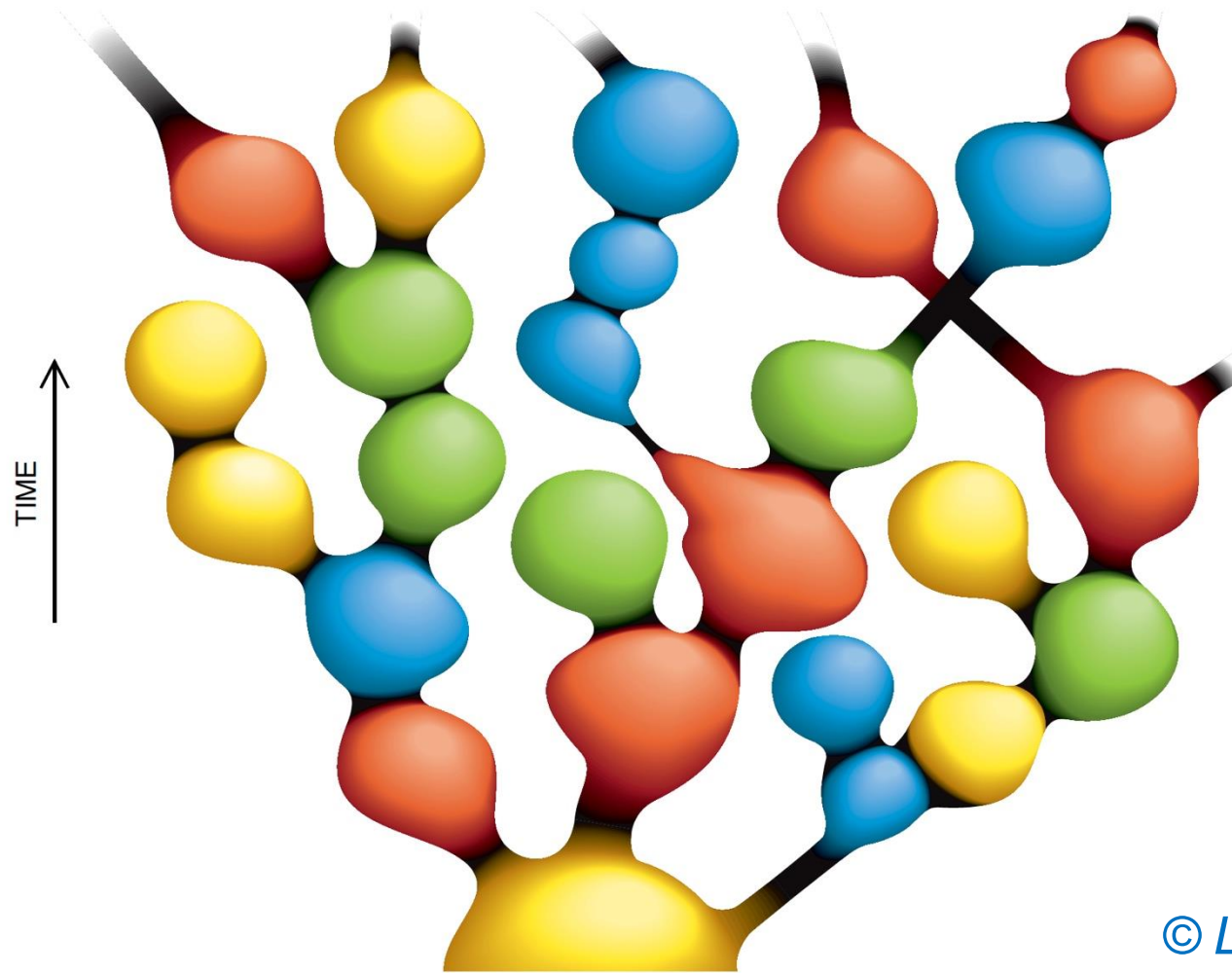
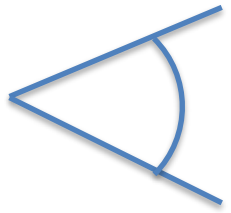
# Outline

1. Why bother with QC?
2. Brief Review of the semi-classical no-boundary wave function  
(+comments)
3. The Observer -> resolving measure problem  
(+comments)
4. Towards a microscopic formulation ...  
(+comments, perhaps)

# The multiverse in the late 1990s....



# The multiverse in the late 1990s....



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“Anthropic Arguments in Fundamental Physics”, Cambridge, Sep 2001

## Wave function of the Universe

J. B. Hartle

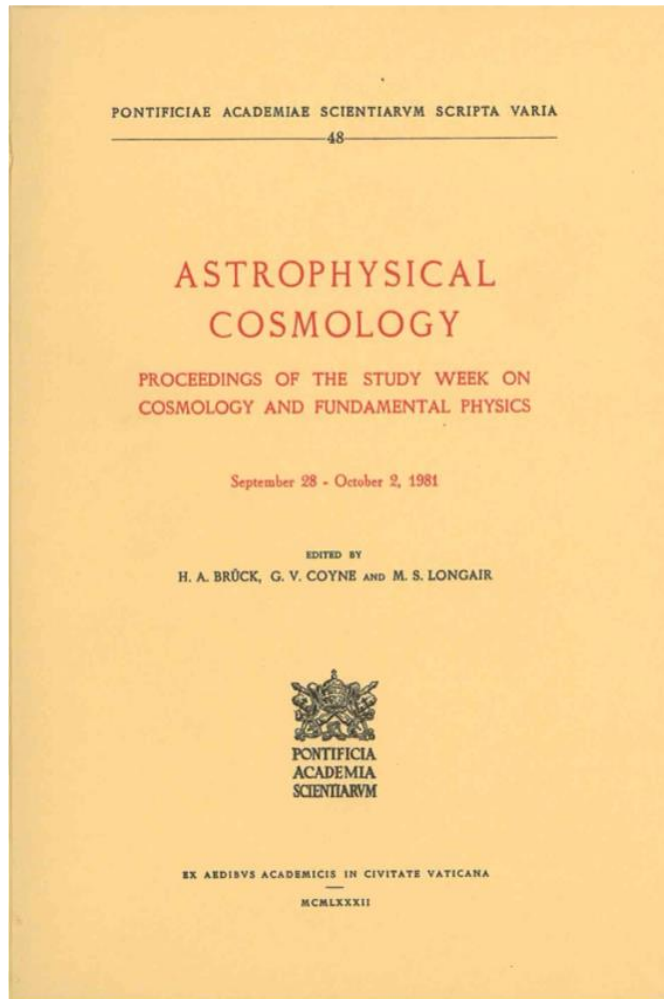
*Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637  
and Institute for Theoretical Physics, University of California, Santa Barbara, California 93106*

S. W. Hawking

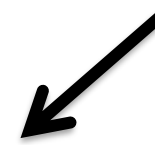
*Department of Applied Mathematics and Theoretical Physics, Silver Street, Cambridge, England  
and Institute for Theoretical Physics, University of California, Santa Barbara, California 93106*

(Received 29 July 1983)

The quantum state of a spatially closed universe can be described by a wave function which is a functional on the geometries of compact three-manifolds and on the values of the matter fields on these manifolds. The wave function obeys the Wheeler-DeWitt second-order functional differential equation. We put forward a proposal for the wave function of the "ground state" or state of minimum excitation: the ground-state amplitude for a three-geometry is given by a path integral over all compact positive-definite four-geometries which have the three-geometry as a boundary.



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*"The boundary condition of the universe is that it has no boundary."*

## no-boundary wave function

Semi-classically:  $\Psi [{}^3g, \phi] \approx A \exp(iS)$

When  $|\nabla A| \ll |\nabla S| \longrightarrow$  boundary configuration  $({}^3g, \phi)$   
evolves classically

Then, WDW eq implies  $p_q = \nabla_q S$

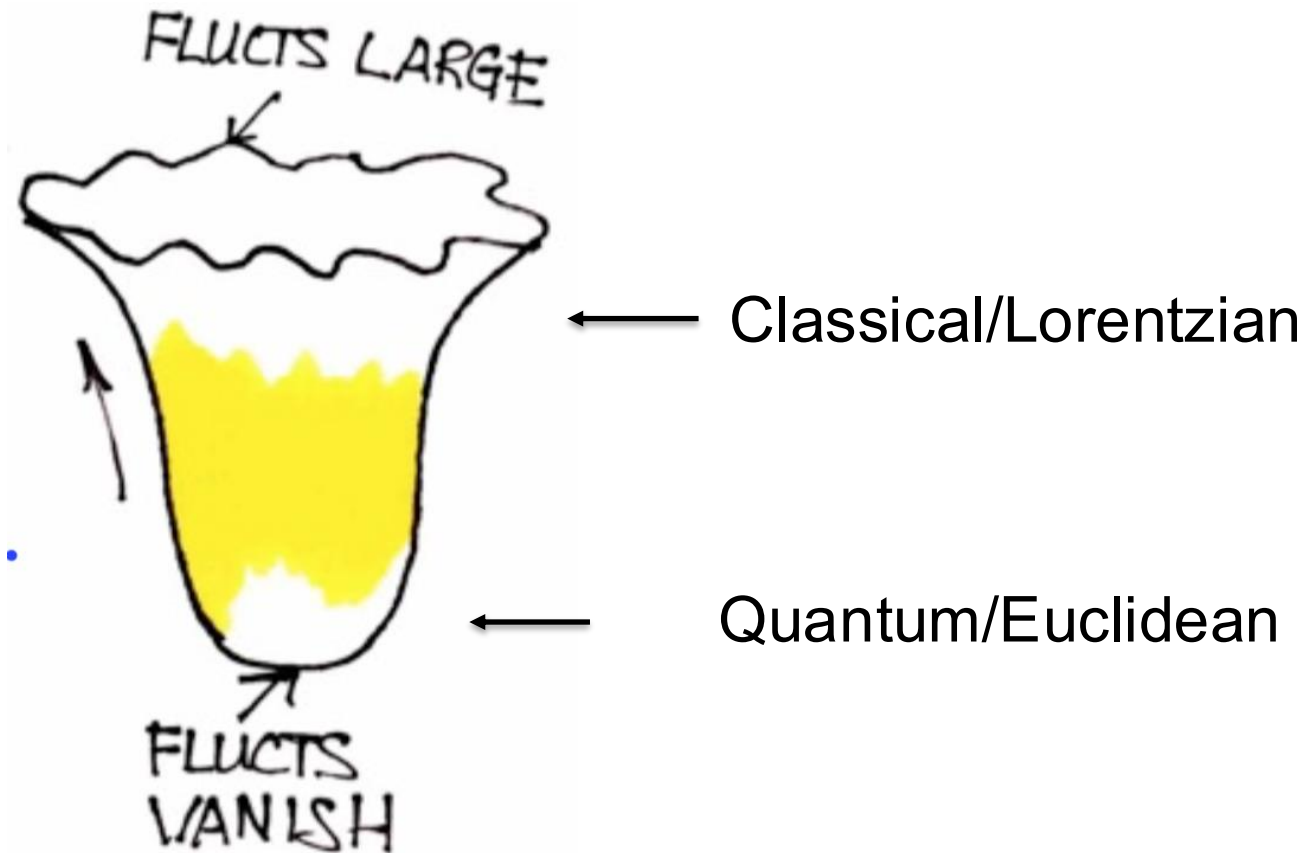
$$P_{hist} \propto A^2 = \exp(-2I_R/\hbar)$$

# no-boundary wave function

[Halliwell, Hawking, Hartle, TH, Laflamme, Louko, Luttrell, Lyons, Moss, Shellard, ...]

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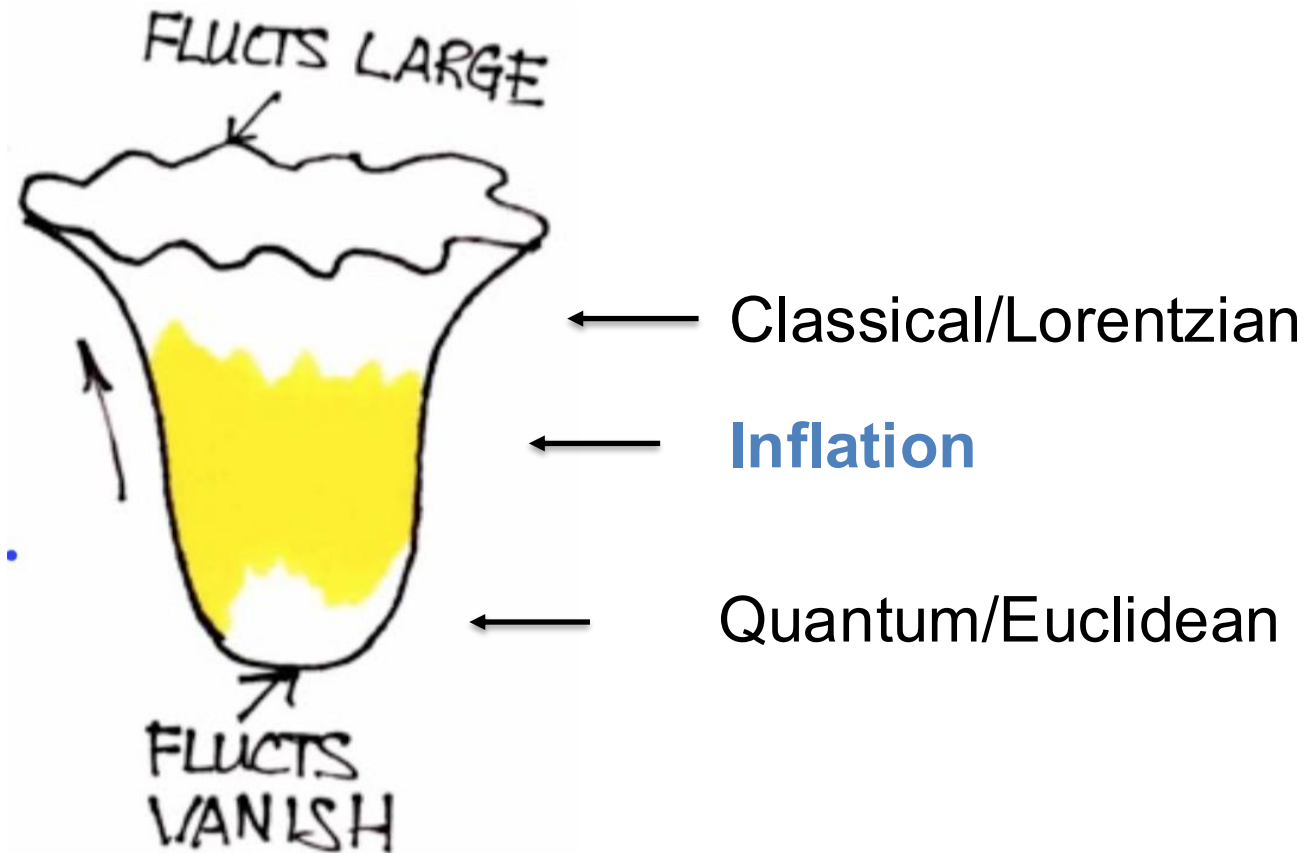


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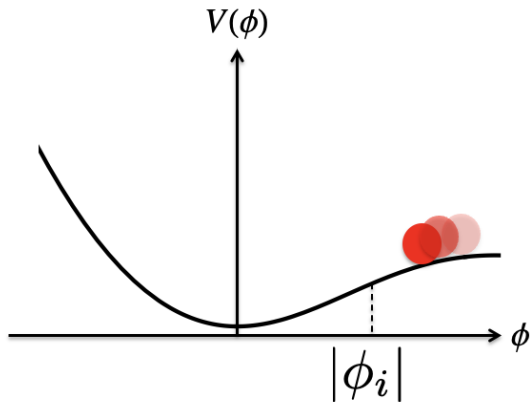
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# no-boundary wave function

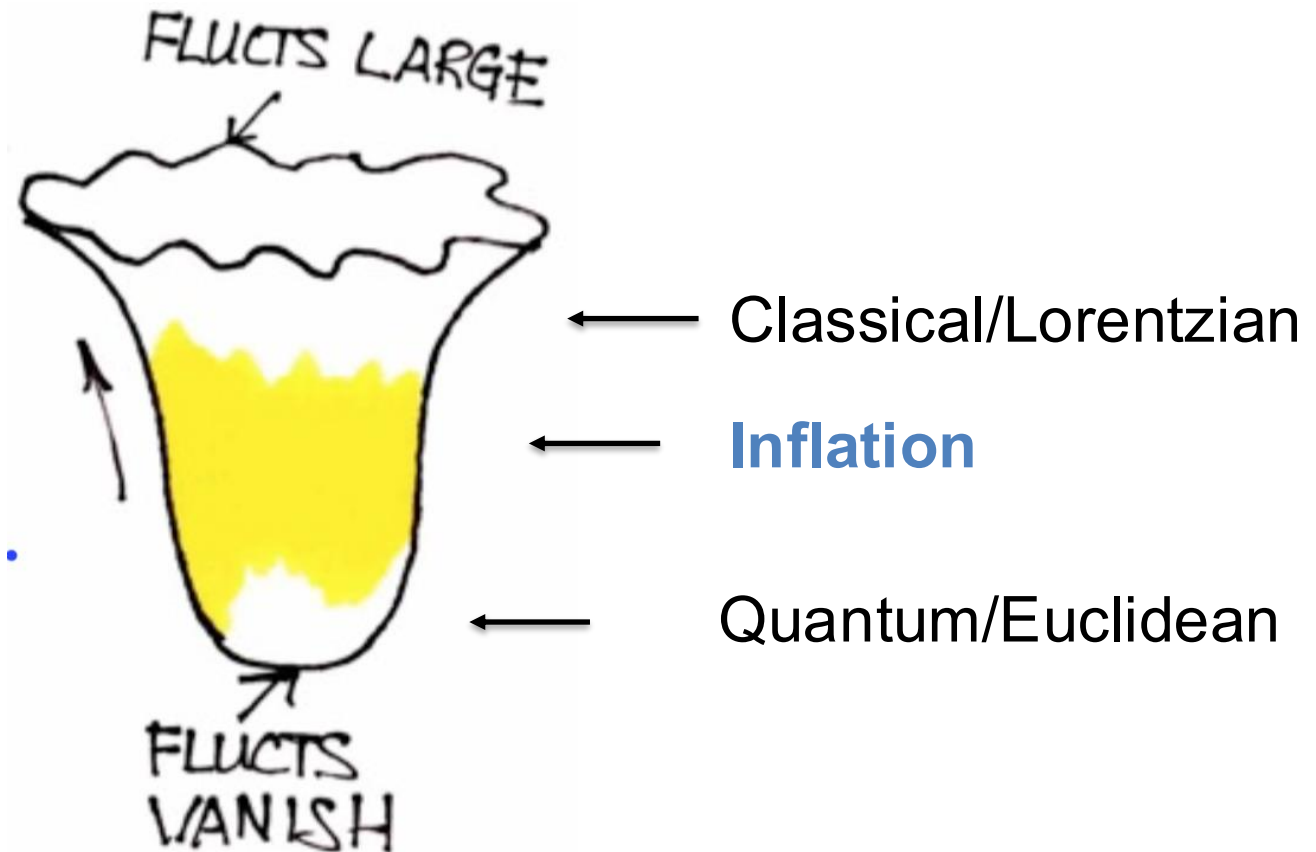
Semi-classically:

$$\Psi[{}^3g, \phi] \approx A \exp(iS)$$



Regularity leaves  $|\phi_i|$  free:

- ensemble of inflationary universes
- with fluctuations initially in BD
- leading to a physical arrow of time



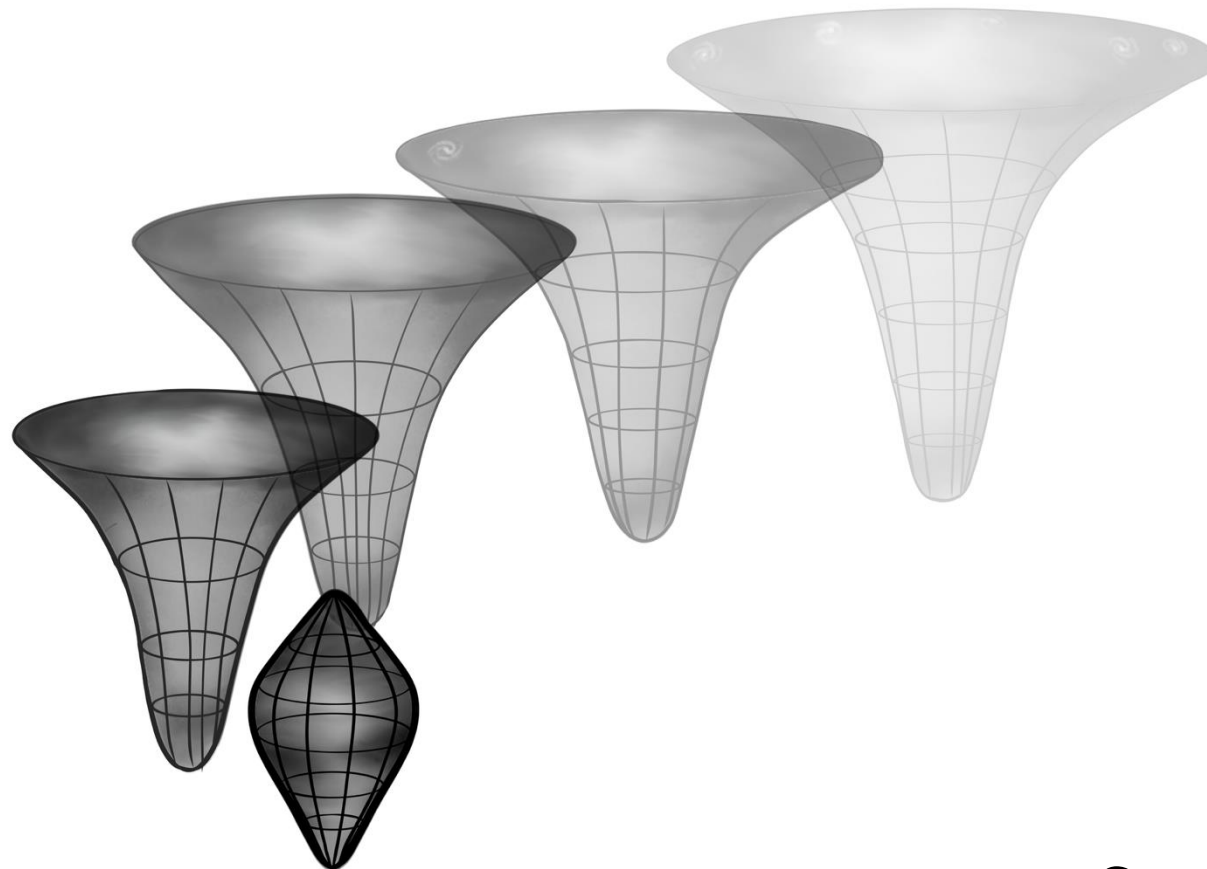
no-boundary wave function

Semi-classically:  $\Psi [{}^3g, \phi] \approx A \exp(iS)$

$$|\Psi_{HH}|^2 \sim e^{1/V(|\phi_i|)} \prod_n e^{-\alpha_n \zeta_n^2}$$

# The big puzzle ...

Hartle, Hawking, TH 2011;  
Maldacena 2403.10510



$$|\Psi|^2 \sim e^{1/V(\phi_i)}$$

Comment 1: no-boundary wave function + KSW ?

[TH, Janssen, Karlsson, 2023]

**Kontsevich- Segal:** complex metrics qualify as backgrounds for physically meaningful QFTs if

$$\text{Re} \left( \sqrt{g} g^{\mu_1 \nu_1} \dots g^{\mu_p \nu_p} F_{\mu_1 \dots \mu_p} F_{\nu_1 \dots \nu_p} \right) > 0$$

**Witten:** elevate this to a selection principle of saddles of gravitational path integrals.

**Some evidence:** KSW criterion eliminates pathological wormholes, but it allows e.g. complexified black hole solutions.

# no-boundary wave function + KSW ?

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**Some evidence:** KSW criterion eliminates pathological wormholes, but it allows e.g. complexified black hole solutions.

Do the fuzzy no-boundary instantons that describe the origin of inflation satisfy the KSW criterion?

# KSW selection

What can go wrong?

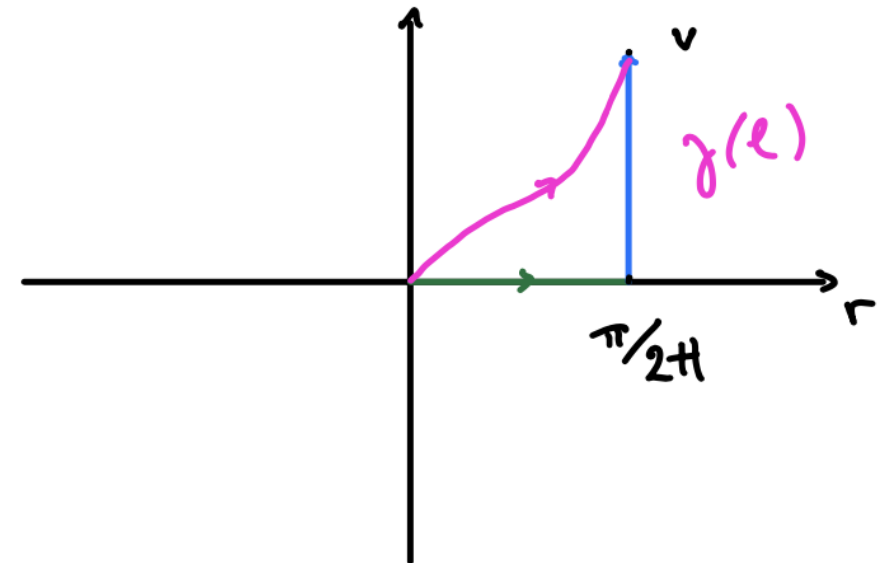
Consider e.g. the  $p=0$  criterion:

$$\operatorname{Re} \sqrt{\det g(x)} > 0$$

In diagonal form this implies

$$\sum_{\mu=1}^D |\arg \lambda_{\mu}(x)| < \pi$$

where  $g_{\mu\nu}(x) = \lambda_{\mu}(x) \delta_{\mu\nu}$



# KSW selection

#	$V/\Lambda$	allowable	disallowable
①	$1 + \cos(\phi/f)$	$[2, 6.09)$	$[6.09, 10]$
②	$1 - \phi^2/\mu^2$	$[10^{1/2}, 10^4]$	
③	$1 - \phi^4/\mu^4$	$[10^{-1}, 10^2]$	
④	$1 - \exp(-q\phi)$	$[10^{-3}, 10^3]$	
⑤	$1 - \mu^2/\phi^2$	$[10^{-6}, 10^3]$	
⑥	$1 + \alpha \log \phi$	$[10^{-3}, 10]$	
⑦	$[1 - \exp(-\sqrt{2}\phi/\sqrt{3\alpha})]^2$	$[10^{-1}, 93.9)$	$[93.9, 10^4]$
⑧	$\phi^p$	$[1/2, 1.05)$	$[1.05, 4]$

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The KSW criterion selects those no-boundary saddles in which the universe emerges on a concave patch of the scalar slow-roll potential

→ stronger theoretical prior

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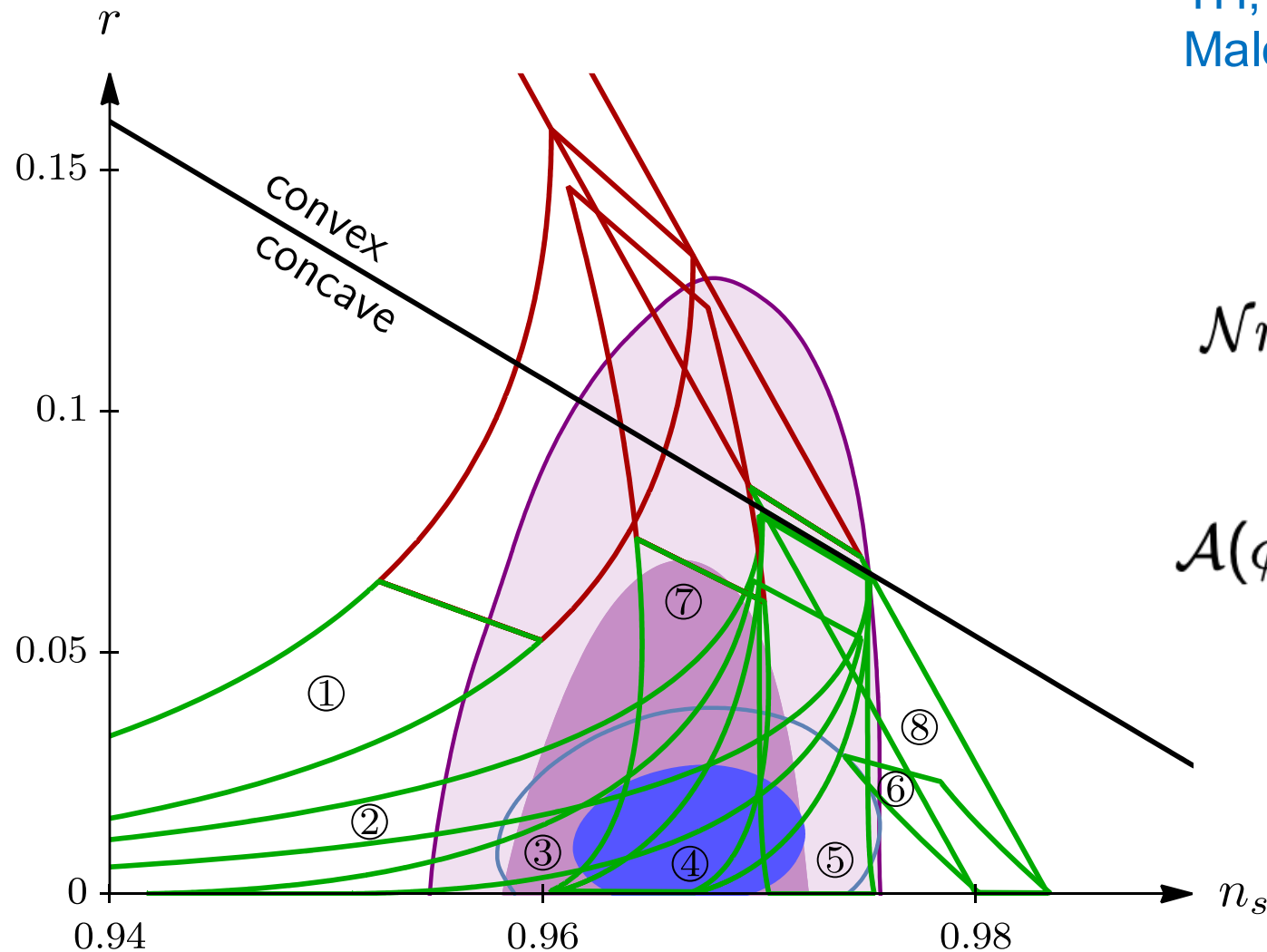
The KSW criterion selects those no-boundary saddles in which the universe emerges on a concave patch of the scalar slow-roll potential

→ stronger theoretical prior

Comment 2: → add also swampland constraints? I don't think so ...

# Observational implications?

TH, O. Janssen, J. Karlsson, 2023;  
Maldacena 2024; Janssen 2024

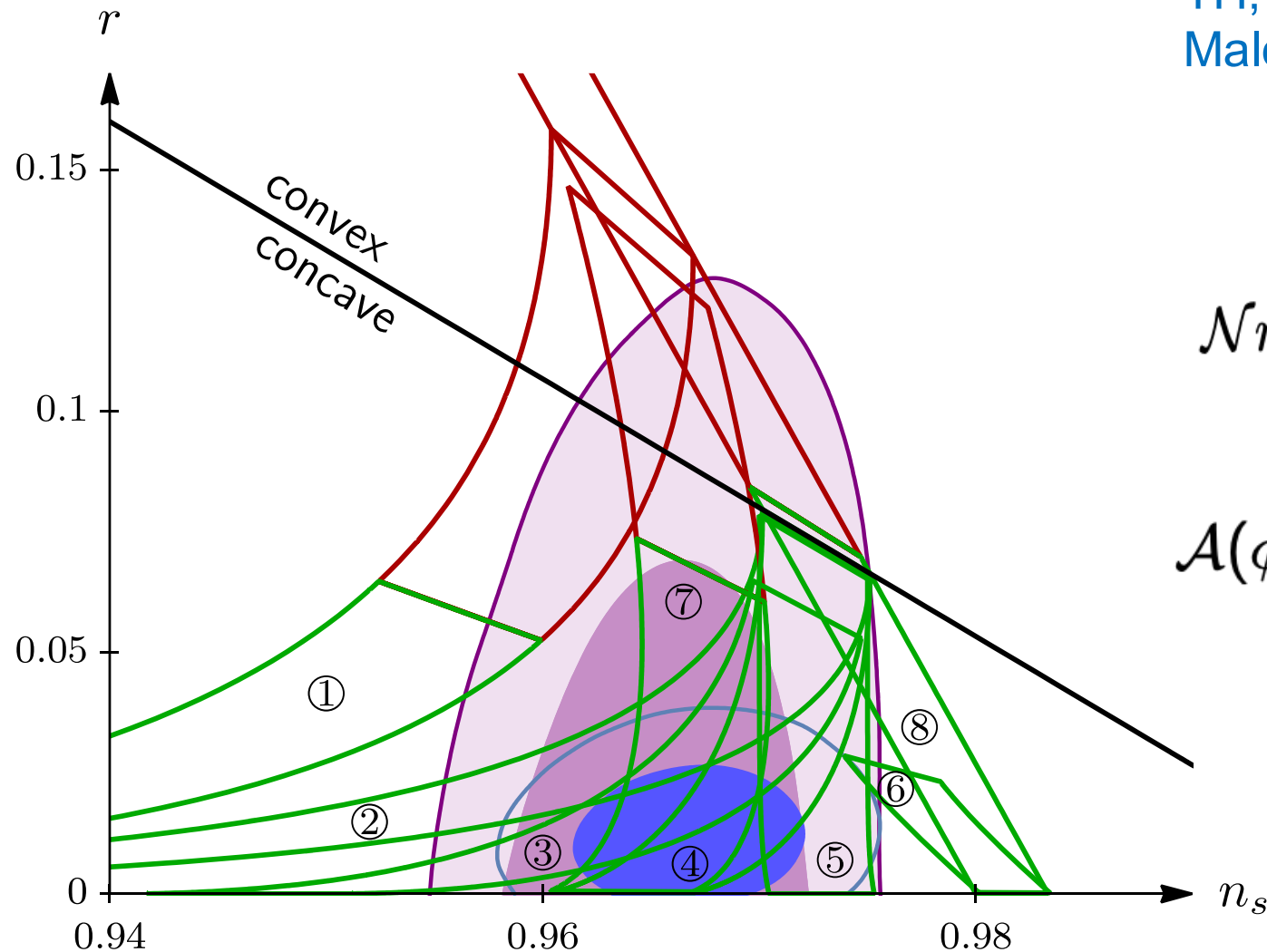


$$\mathcal{N}r \leq 8, \quad \text{or} \quad r \leq \frac{8}{\mathcal{N}} \sim 0.13$$

$$\mathcal{A}(\phi_*, \chi) \equiv \frac{V'_*}{V_*} \int_{\phi_*}^{\chi} |d\phi| \frac{V'_*}{V'(\phi)} < 1$$

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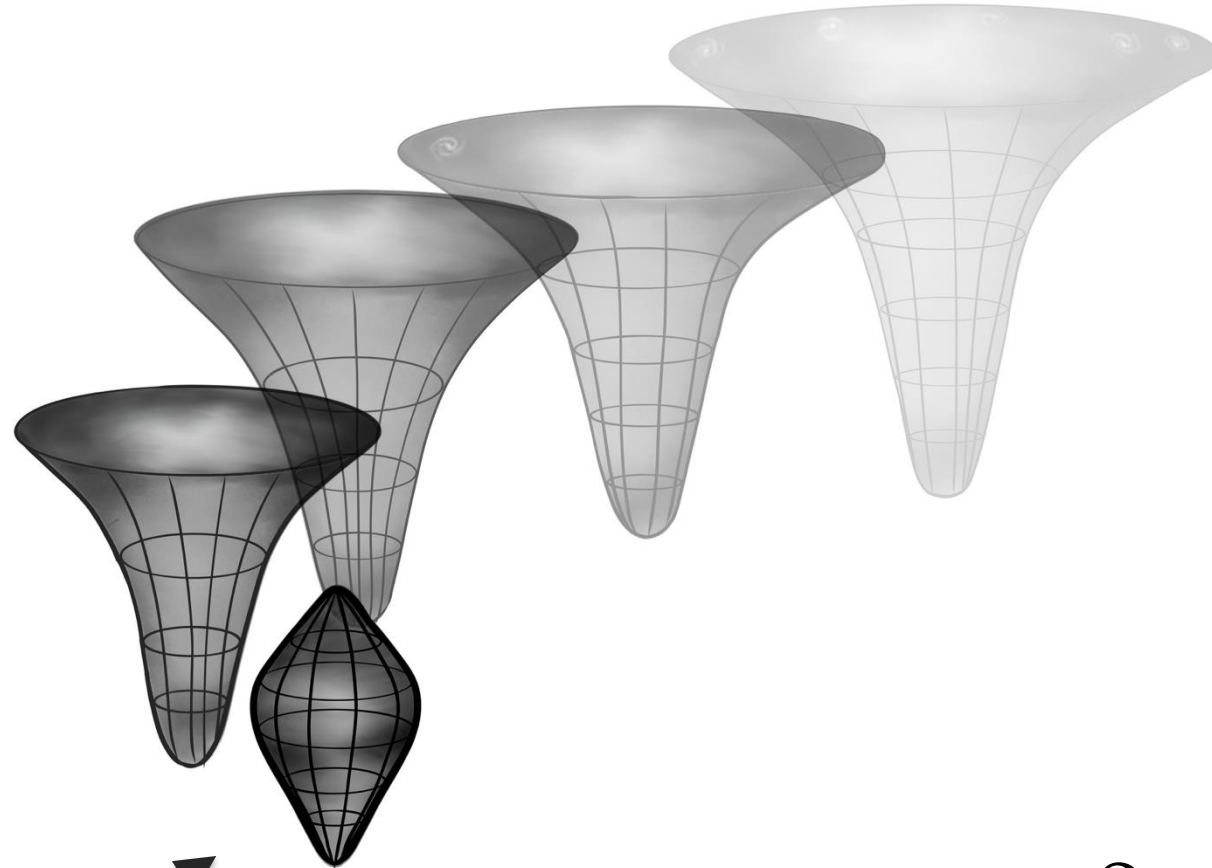
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Comment 3: more tuning gets you out ...

# The big puzzle ...

[Hartle, Hawking, TH 2008;  
Maldacena 2403.10510]

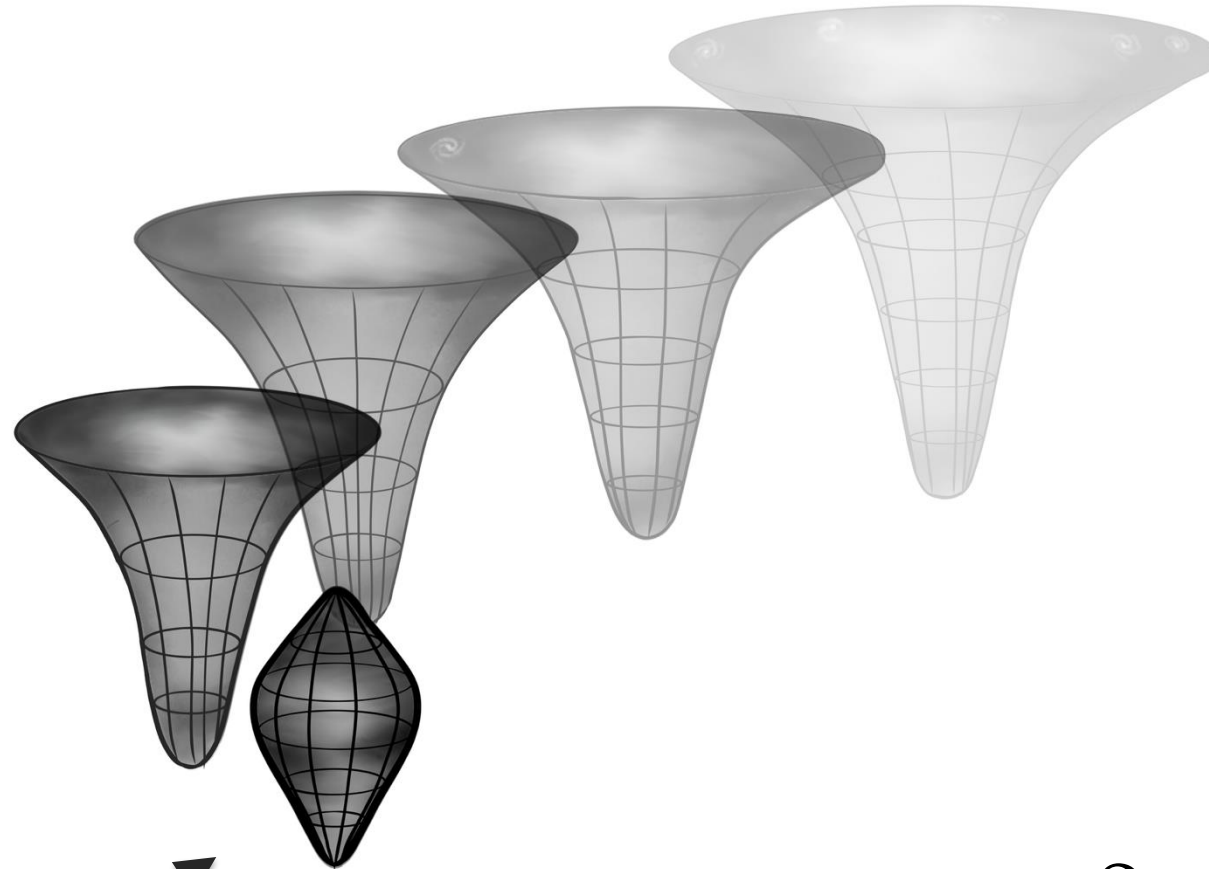
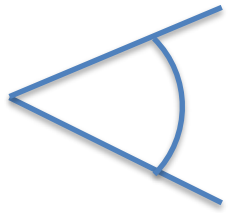


Brief burst  
of inflation

$$|\Psi|^2 \sim e^{1/V(\phi_i)}$$

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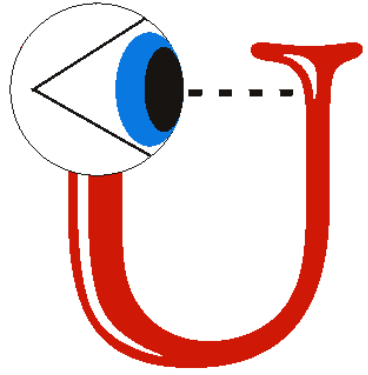


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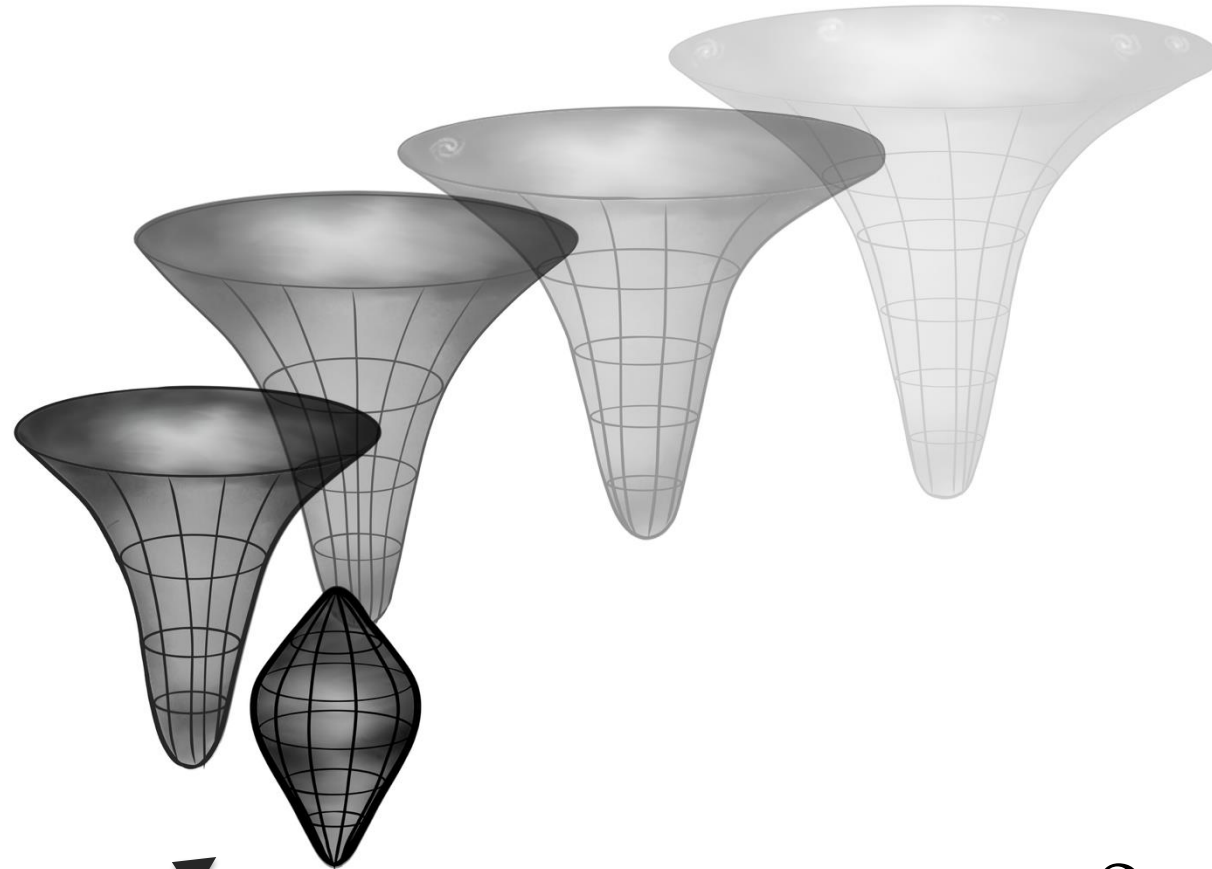
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© Wheeler

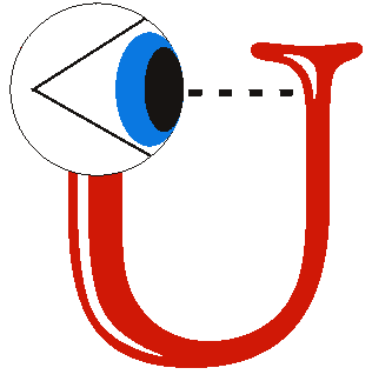


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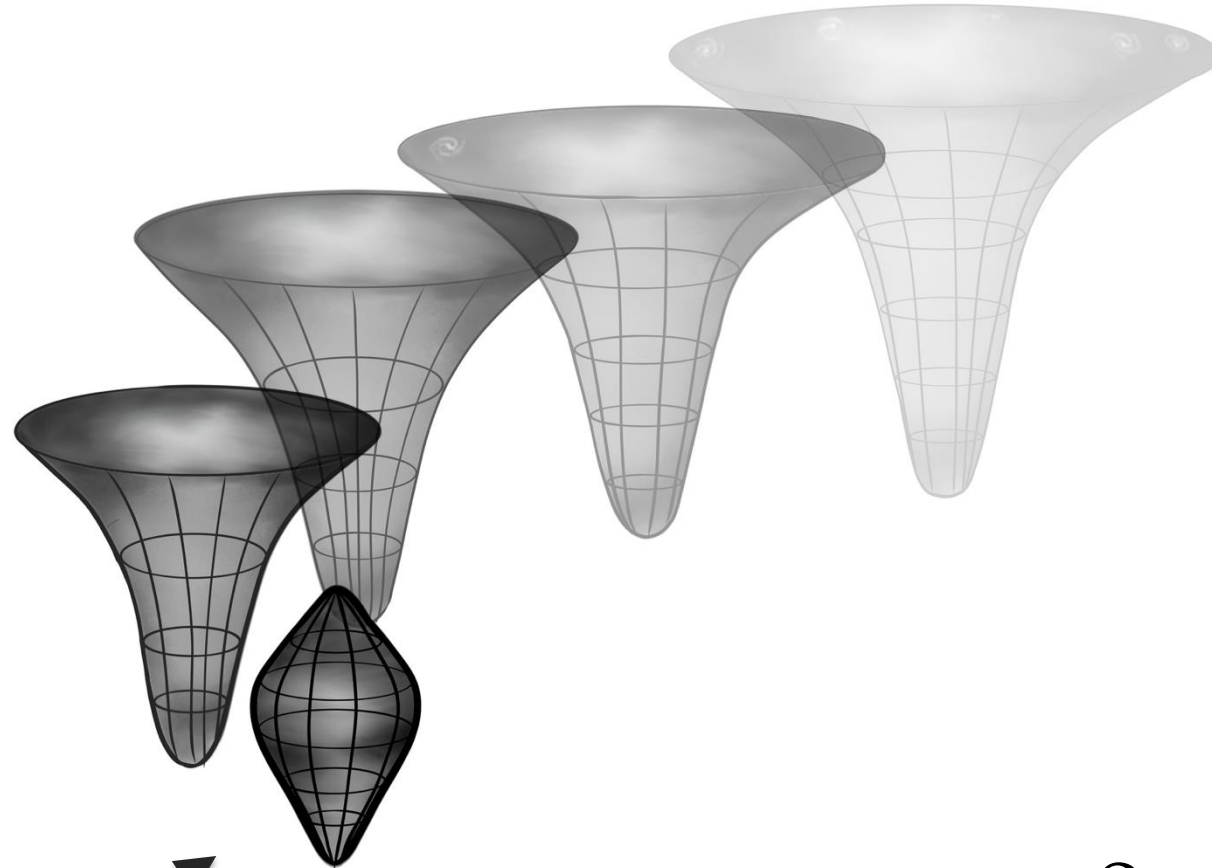
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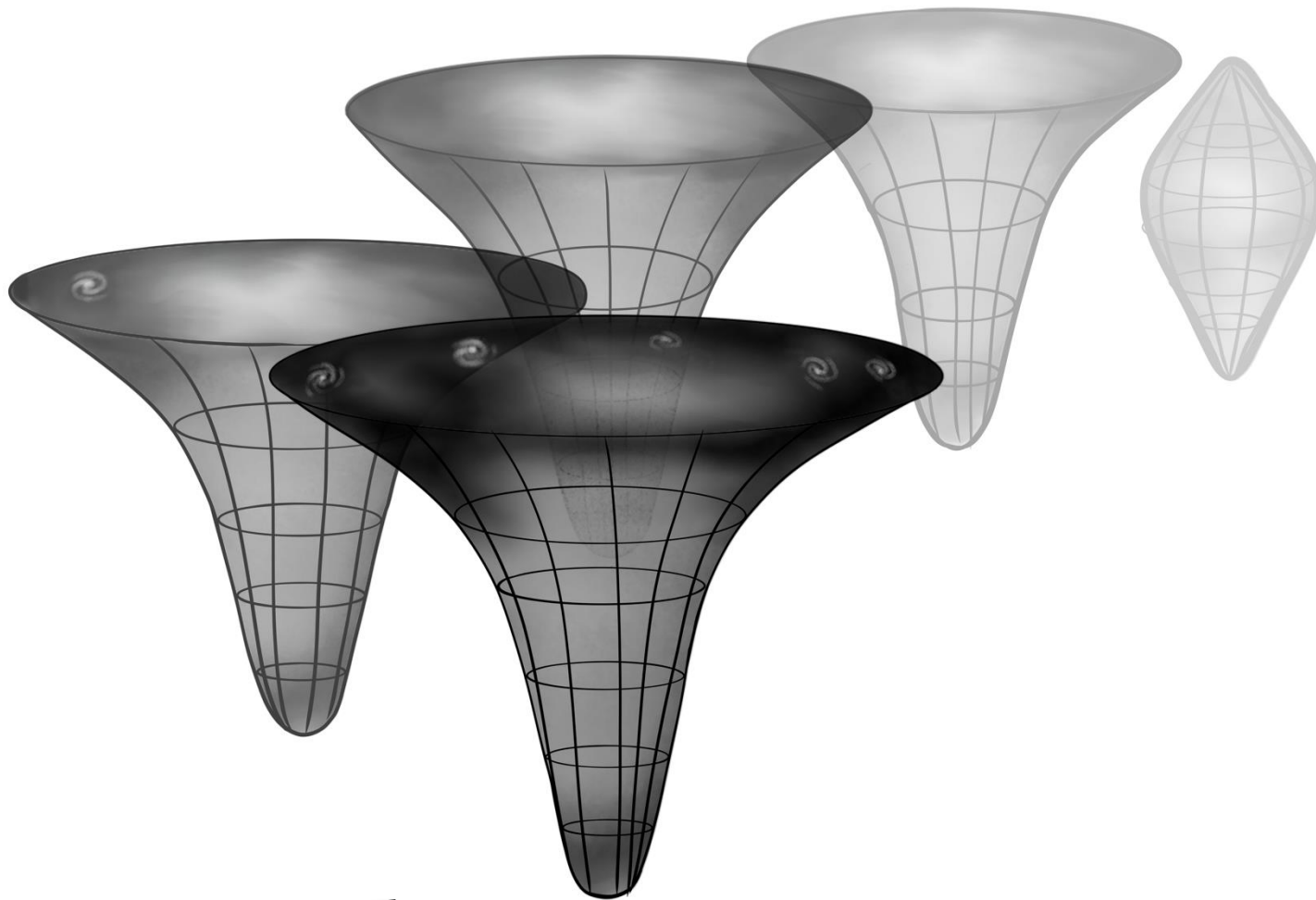
Brief burst  
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“Observer”



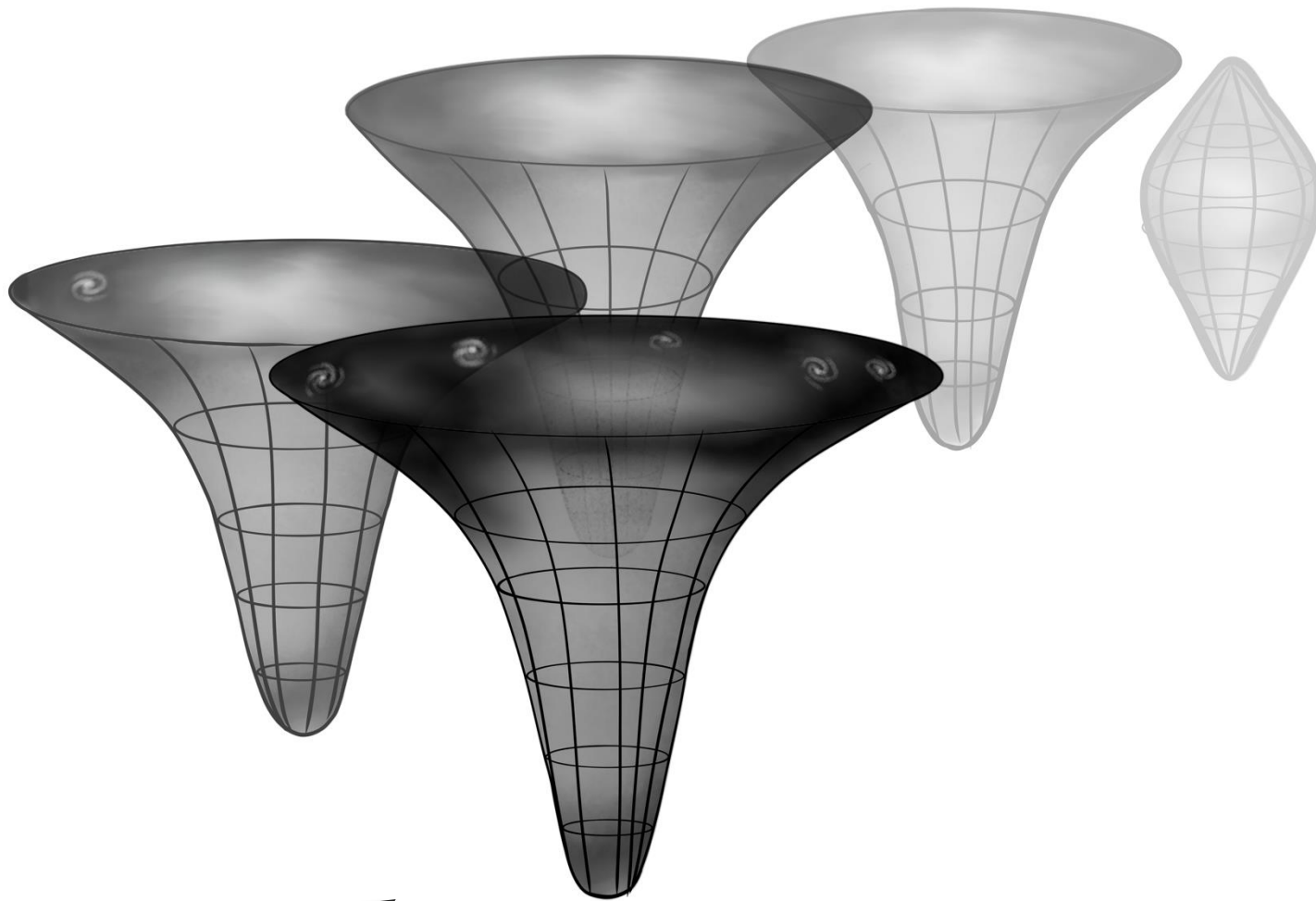
$$P(N | D \geq 1, \Psi)$$

$$|\Psi|^2 \sim e^{1/V(\phi_i)}$$



Strong burst  
of inflation

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Strong burst  
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$$|\Psi_{fl}|^2 \sim \prod_n e^{-\alpha_n \zeta_n^2}$$



$$P(N|D \geq 1, \Psi)$$


# no-boundary measure 2.0

$$P(N|D^{\geq 1}) \sim \left(1 - [1 - p_H(D)]^{N_H(N)}\right) \exp[3\pi/m^2 N]$$

$|\Psi|^2$



$|\Psi_{fl}|^2$




- Simple observer  $D \rightarrow$  low  $N$  saddle dominates  $\rightarrow$  'Boltzmann Brain' in de Sitter


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- Refined observer  $D \rightarrow$  large  $N$  saddle can dominate  $\rightarrow$  inflationary seeds of structure

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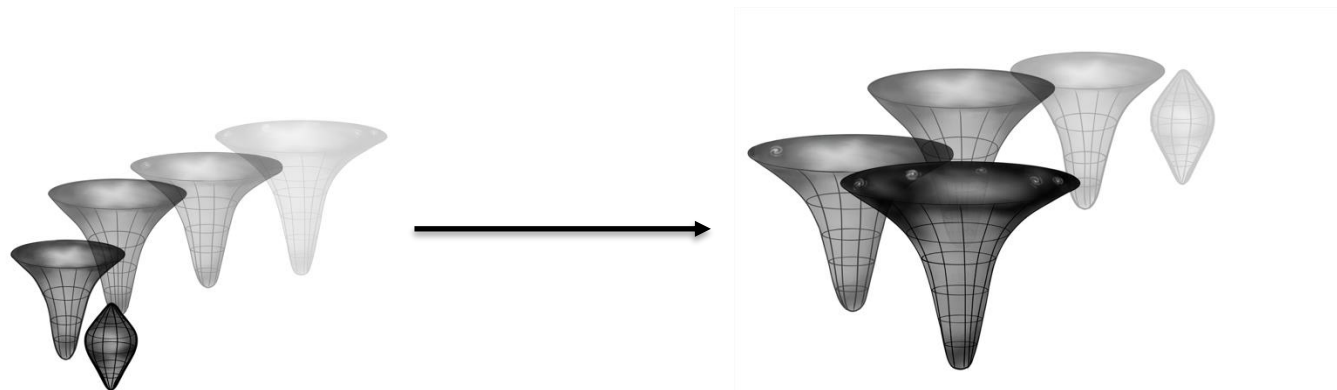
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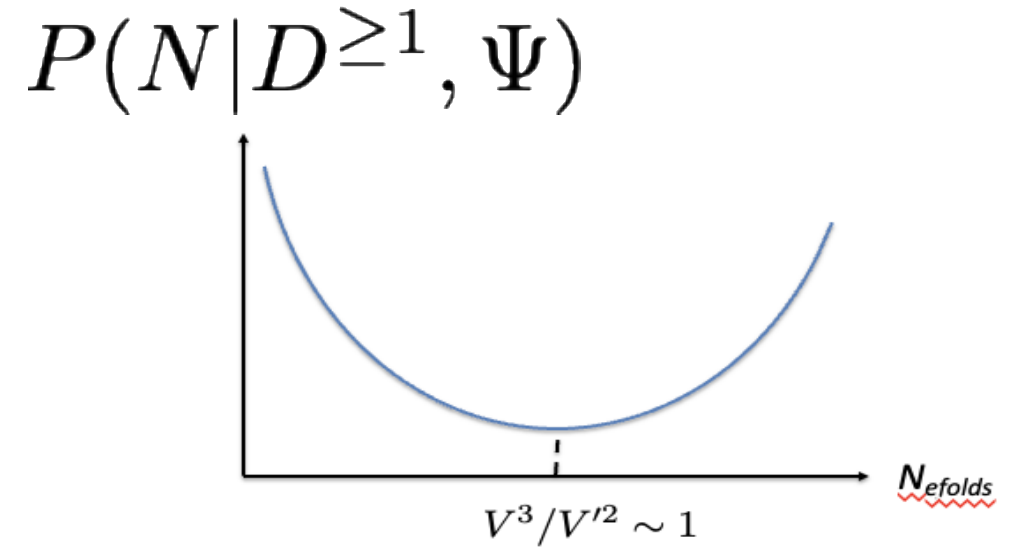
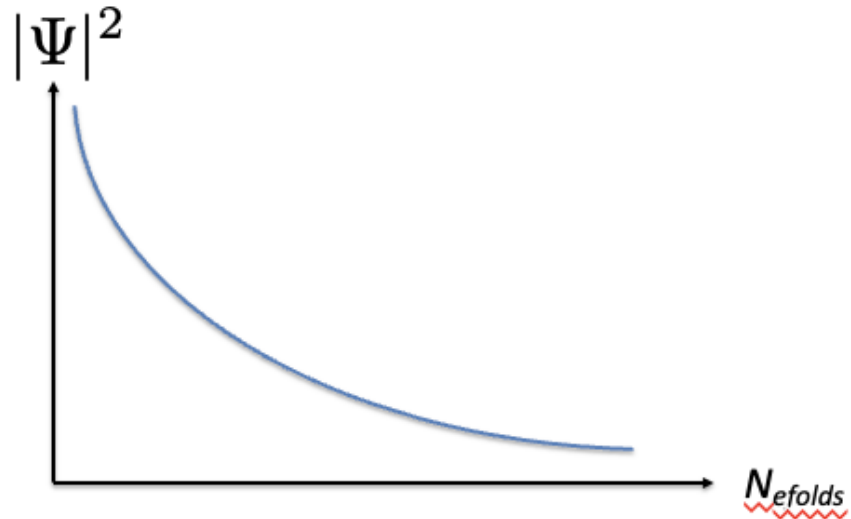
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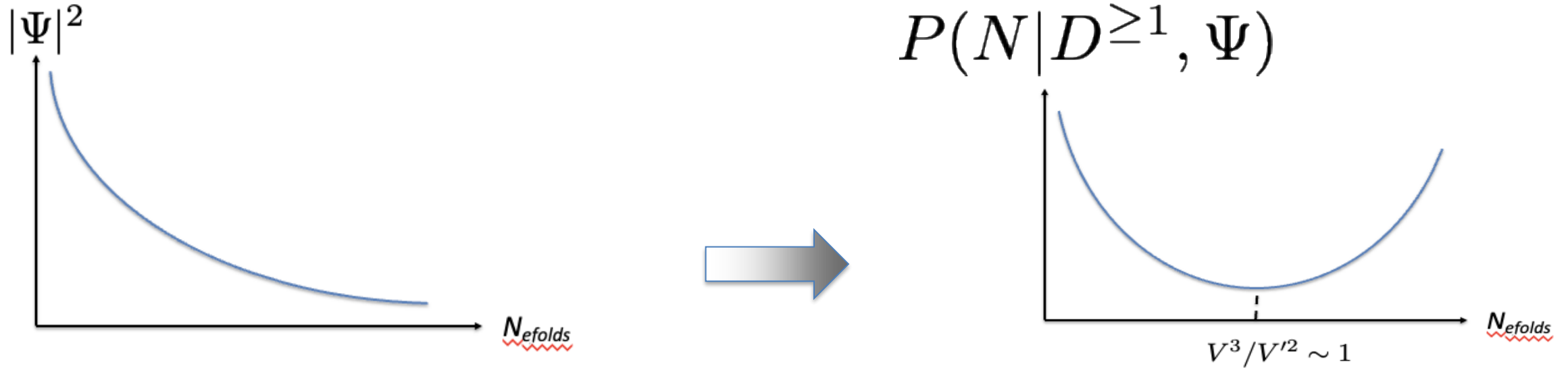
Distribution exhibits a Page-like transition:



**Comment 1:** Transition occurs always at the threshold of eternal inflation:

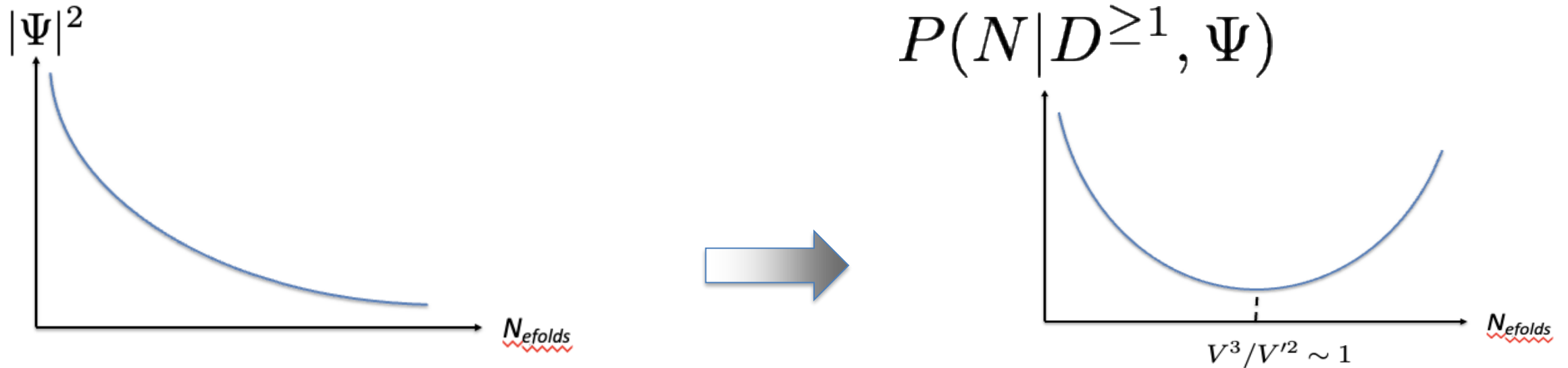


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**Comment 2:** Replication of D regulates observer-weighting

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**Comment 2:** Replication of D regulates observer-weighting

Upshot: In some models, wave function predicts that our universe emerged from a long period of inflation.

## More comments ...

**Comment 3:** For an 'anthropic' observer D this amounts to anthropic reasoning, but without the fatal ambiguities of an anthropic principle external to the theory. Here this is merely quantum mechanics of closed systems... and the data D specifying the 'observer' can have nothing to do with life.

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**Comment 4:** Maldacena lukewarm [2403.10510]

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- GPI ill-defined?
- Hartle-Hawking -> thermal state
- Not in Hartle-Hawking state?
- Tunneling wavefunction?
- Quantum corrections?
- ...

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- GPI ill-defined?
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- ...

**Comment 5:** Perhaps there are exceptions to  $|\Psi|^2 \sim e^{1/V(\phi_i)}$

# What about k-inflation?

Damour et al. 1999  
Creminelli et al. 2006

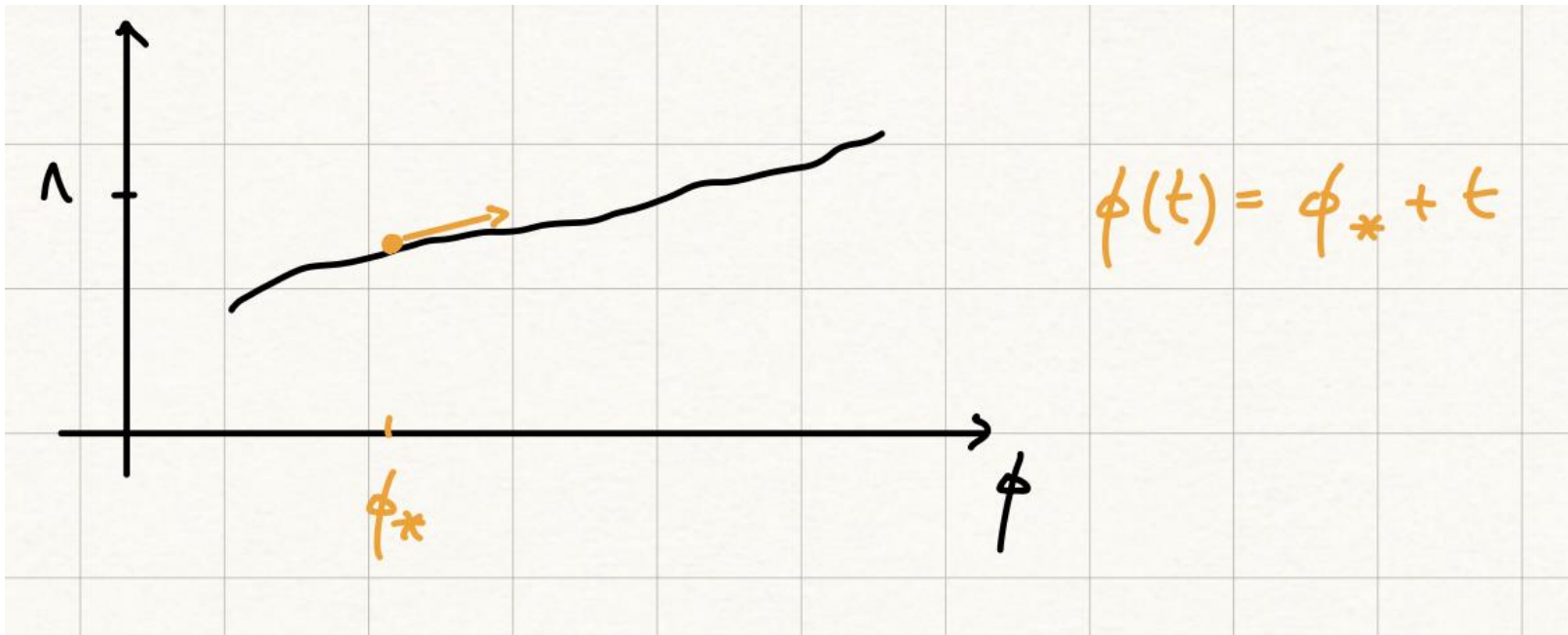
$$S = \int_M d^4x \sqrt{-g} \left( \frac{R}{2} + P(\phi, X) \right) - \int_{\partial M} d^3y \sqrt{h} K,$$

$$X \equiv -\frac{1}{2} (\partial\phi)^2.$$

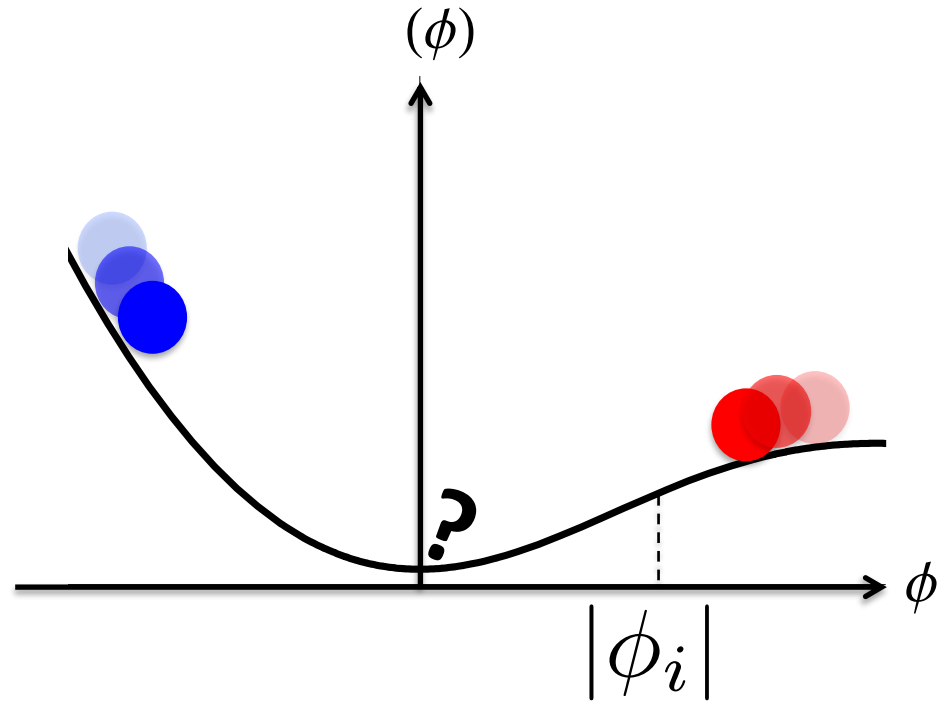
$$|\Psi|^2 \sim \exp\left(\frac{8\pi^2}{H_*^2}\right).$$

$$P(\phi, X) = K(\phi)X + L(\phi)X^2 + \mathcal{O}(X^3) \quad \text{as } X \rightarrow 0$$

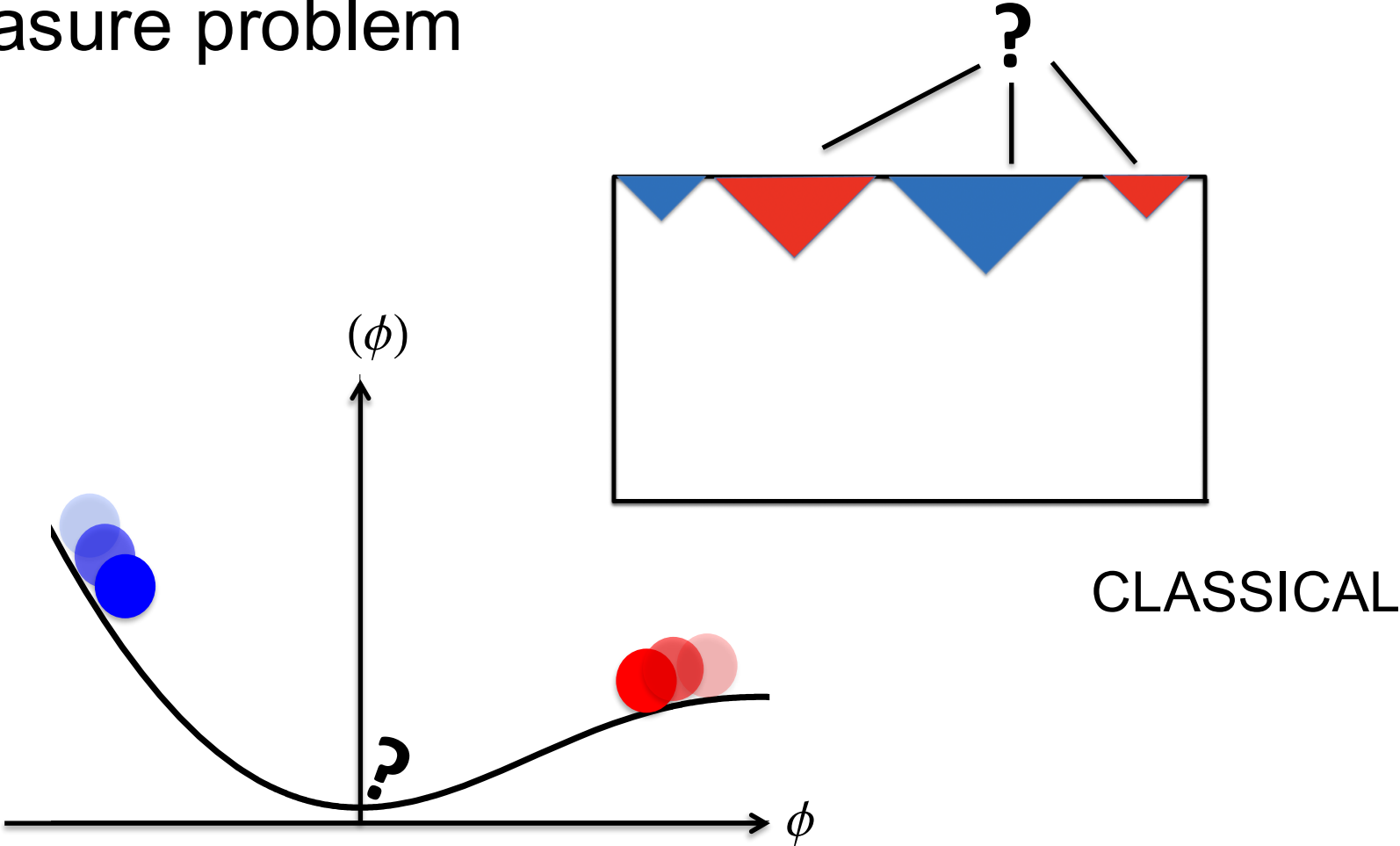
TH, O. Janssen, E. Pajer, to appear



# Resolving the measure problem



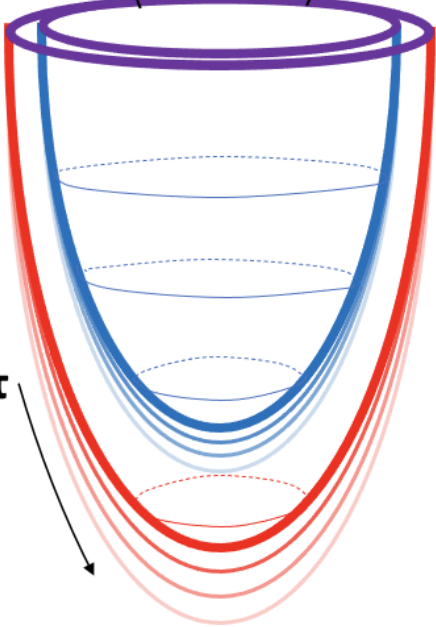
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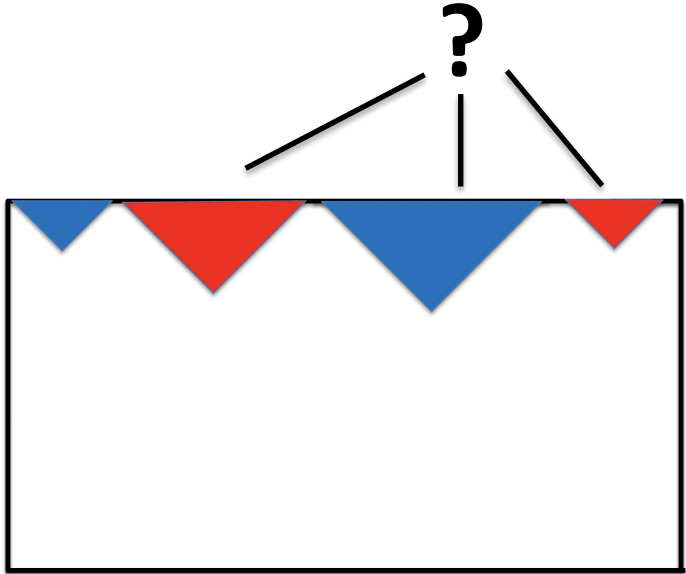
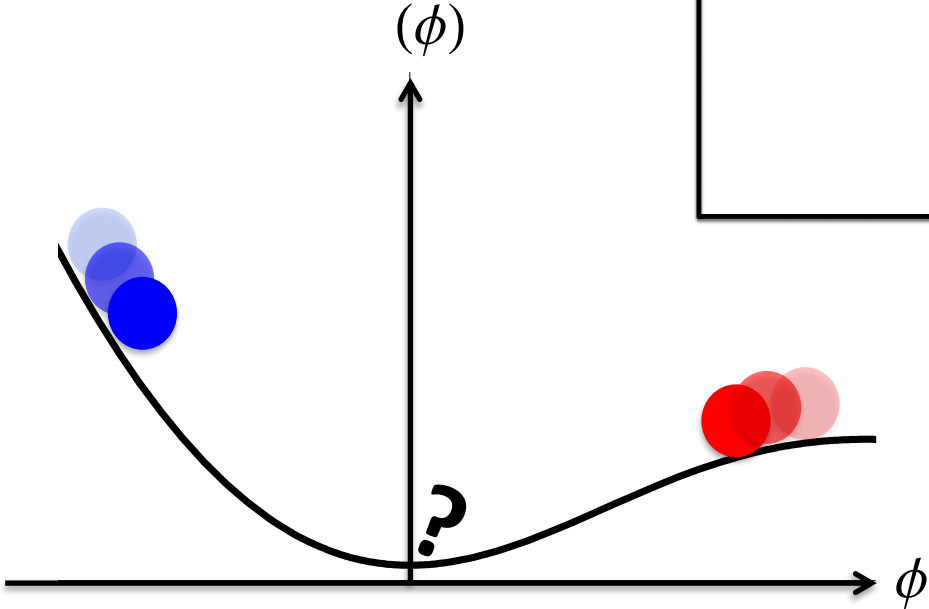
# Resolving the measure problem

$$P(N | D^{\geq 1}, \Psi)$$

*observer*



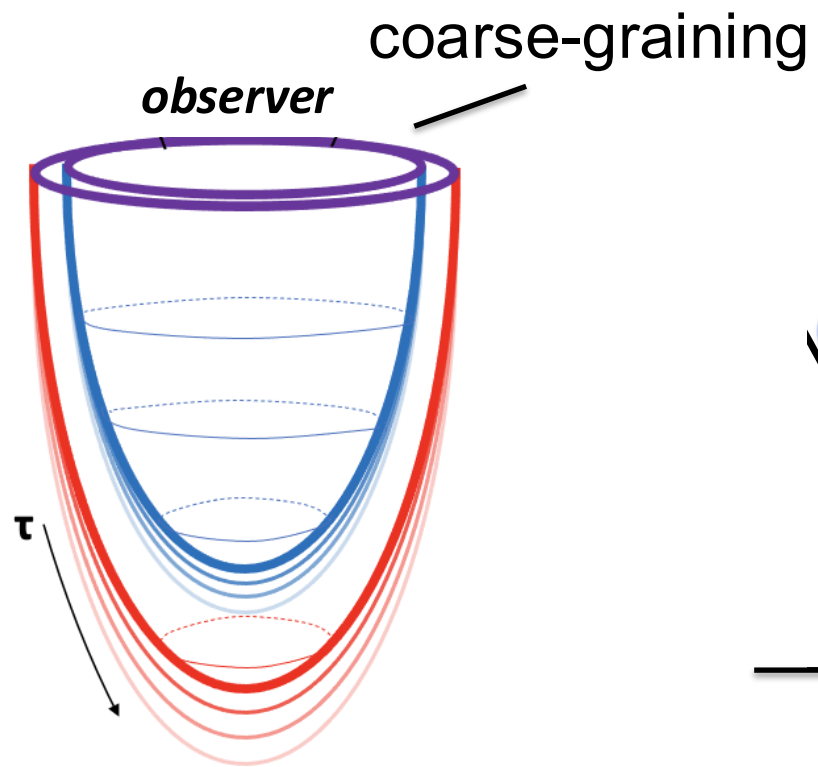
QUANTUM



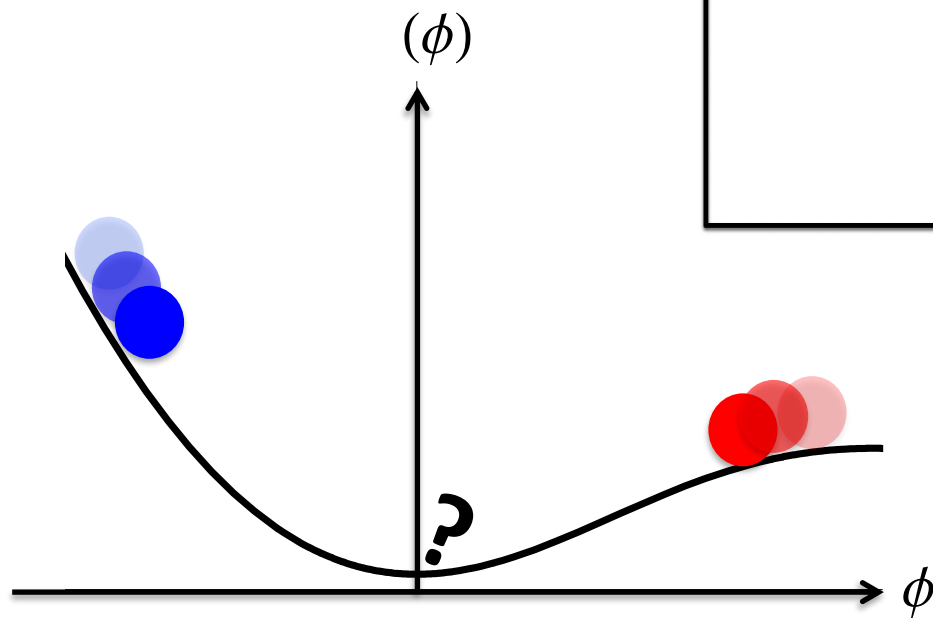
CLASSICAL

# Resolving the measure problem

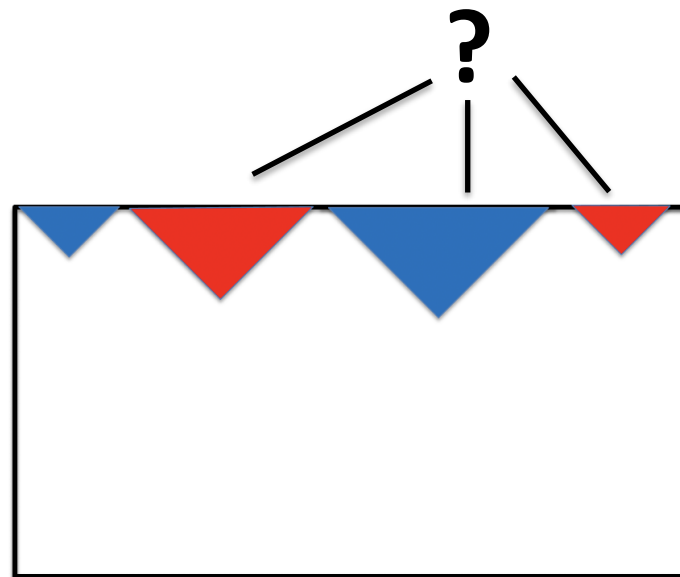
$$P(N | D^{\geq 1}, \Psi)$$



QUANTUM



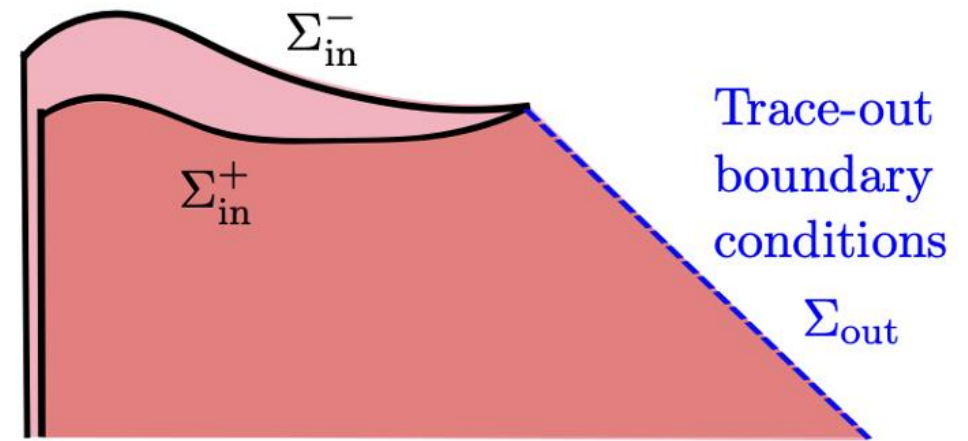
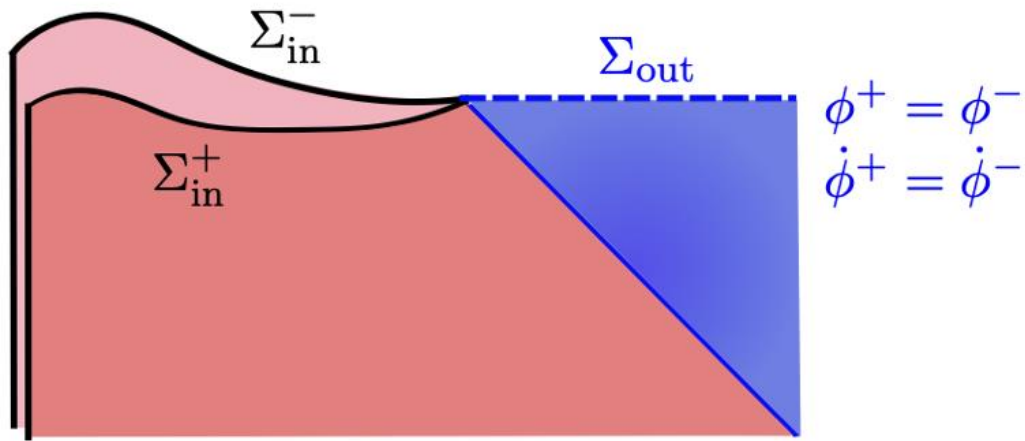
CLASSICAL



# The no boundary density matrix

V. Ivo, Y-Z. Li, J. Maldacena,  
2409.14218;

Hartle, TH 2016,  
*One Bubble to Rule them All*



$$\rho[\Phi_{in}^-(\vec{x}), \Phi_{in}^+(\vec{x})] = \int \mathcal{D}\Phi_{out} \Psi^*[\Phi_{in}^-, \Phi_{out}] \Psi[\Phi_{in}^+, \Phi_{out}] \propto \Psi^*[\Phi_{in}^-, \Phi_{out}^s] \Psi[\Phi_{in}^+, \Phi_{out}^s]$$

Full coarse-graining  $\rightarrow$  usual no-boundary saddle points

## Local Observation in Eternal Inflation

James Hartle,<sup>1</sup> S. W. Hawking,<sup>2</sup> and Thomas Hertog<sup>3</sup>

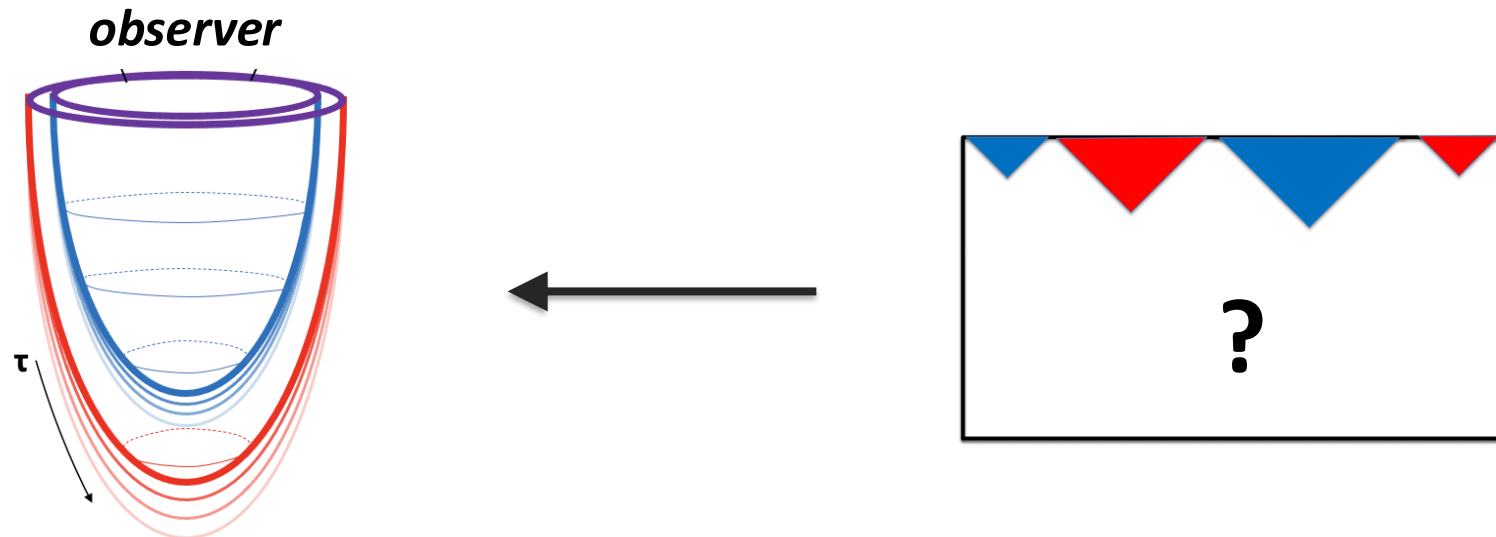
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We consider landscape models that admit several regions where the conditions for eternal inflation hold. It is shown that one can use the no-boundary wave function to calculate small departures from homogeneity within our past light cone despite the possibility of much larger fluctuations on super horizon scales. The dominant contribution comes from the history exiting eternal inflation at the lowest value of the potential. In a class of landscape models this predicts a tensor to scalar ratio of about 10%. In this way the no-boundary wave function defines a measure for the prediction of local cosmological observations.



# Eternal inflation without metaphysics

James Hartle, S.W. Hawking, Thomas Hertog


In the usual account of eternal inflation the universe is supposed to be a de Sitter background in which pocket universes nucleate at a steady rate. However this is metaphysics because there is no way this mosaic structure can be observed. We don't see the whole universe but only a nearly homogeneous region within our past light cone. We show that we can use the no-boundary wave function to calculate small departures from homogeneity within our past light cone despite the possibility of much larger fluctuations on super horizon scales. We find that the dominant contribution comes from the history that exits eternal inflation at the lowest value of the potential and predict, in a certain class of landscape models, a tensor to scalar ratio of about 10%. In this way the no-boundary wave function defines a measure for the prediction of local cosmological observations.

Comments: 4 pages

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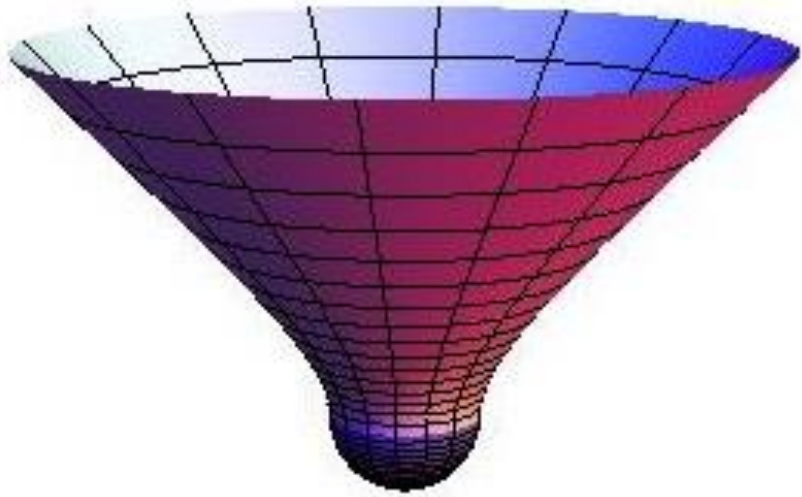
## Submission history

From: Thomas Hertog [[view email](#)]

[v1] Mon, 13 Sep 2010 21:14:56 UTC (9 KB)

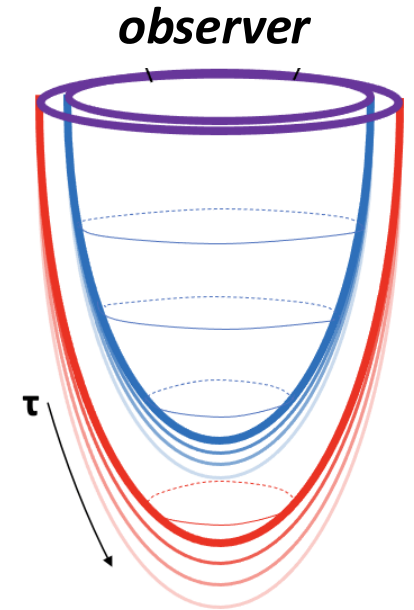
[v2] Sun, 10 Apr 2011 09:17:20 UTC (10 KB)

## Yet another remark



bottom-up cosmology (1983)

“creation from nothing”



top-down cosmology (2011)

“past reconstruction”

We now view the no-boundary proposal not as a probability that the universe started in some way, but as the probability that it ends up looking like it looks at the final surface where we evaluate the wavefunction.

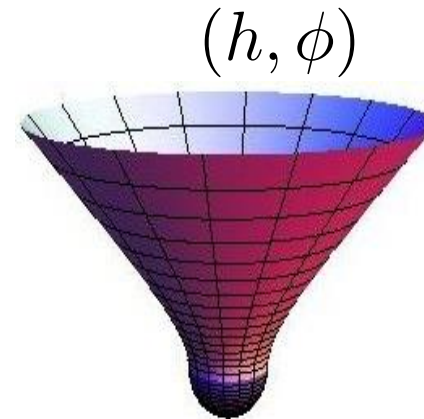
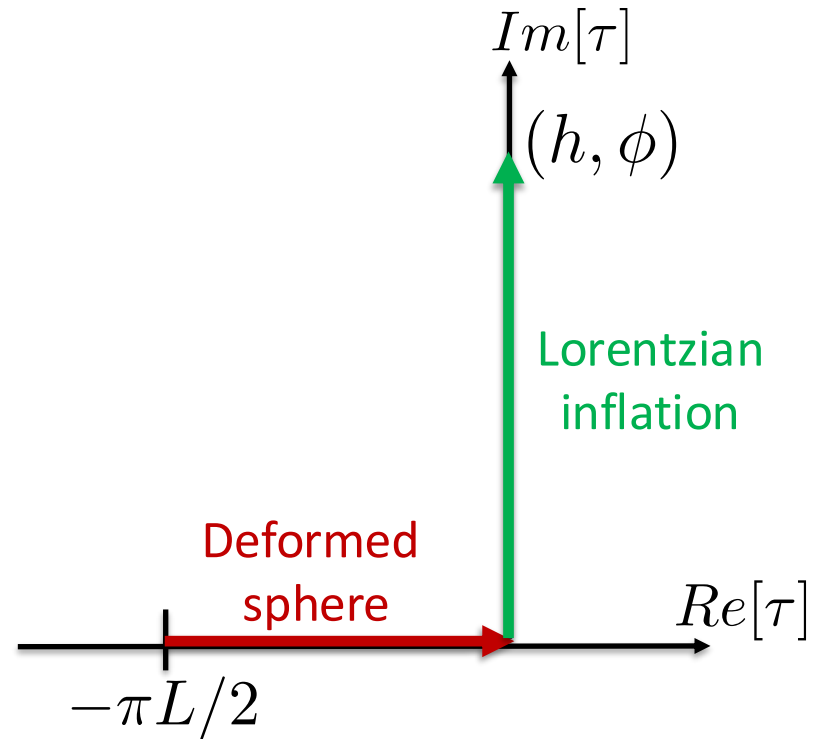
# Outline

1. Why bother with QC?
2. Brief Review of the semi-classical no-boundary wave function  
(+comments)
3. The Observer -> resolving measure problem  
(+comments)
4. Towards a microscopic formulation ...  
(+comments, perhaps)

# Holographic no-boundary measure

[Hartle, TH '11;  
Maldacena '02;  
Harlow, Stanford '11;  
Anninos et al. '12]

$$\Psi_{HH} = \mathcal{A}_{sp} e^{iS}$$

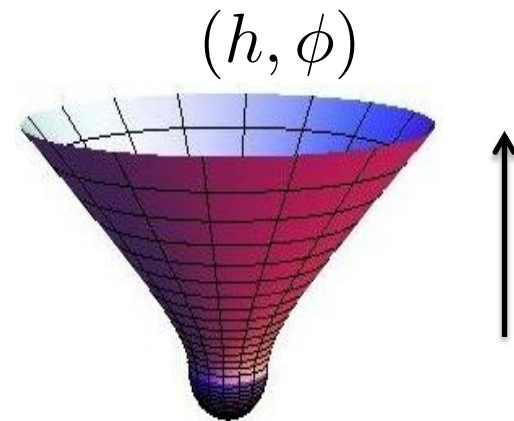
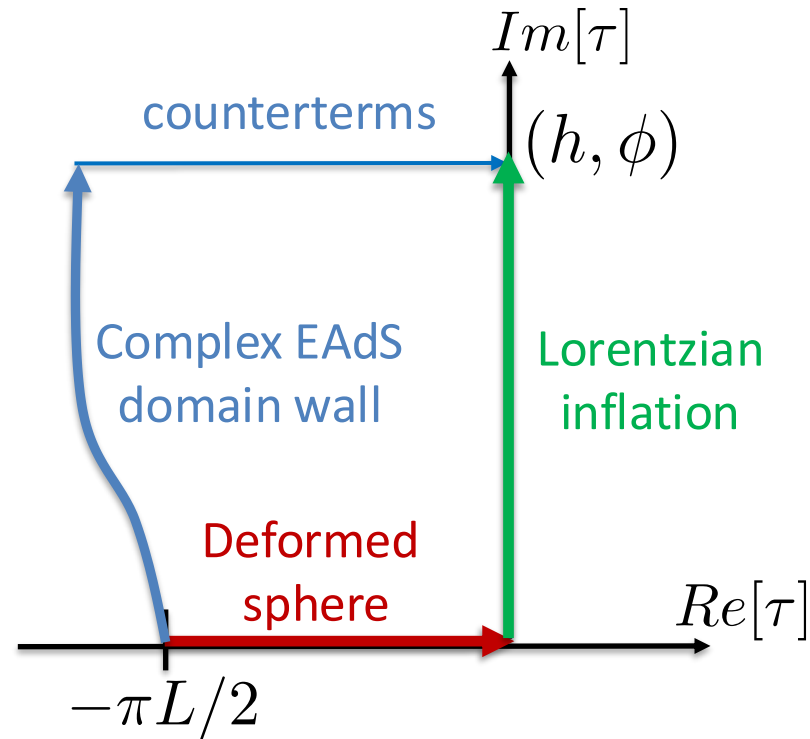


# Holographic no-boundary measure

[Hartle, TH '11;  
Maldacena '02;  
Harlow, Stanford '11;  
Anninos et al. '12]

$$\Psi_{HH} = \mathcal{A}_{sp} e^{iS}$$

$$\log \mathcal{A}_{sp} = I_{asEAdS}^{\text{reg}}$$

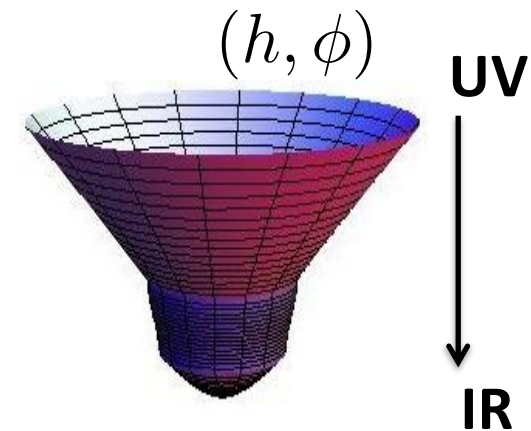
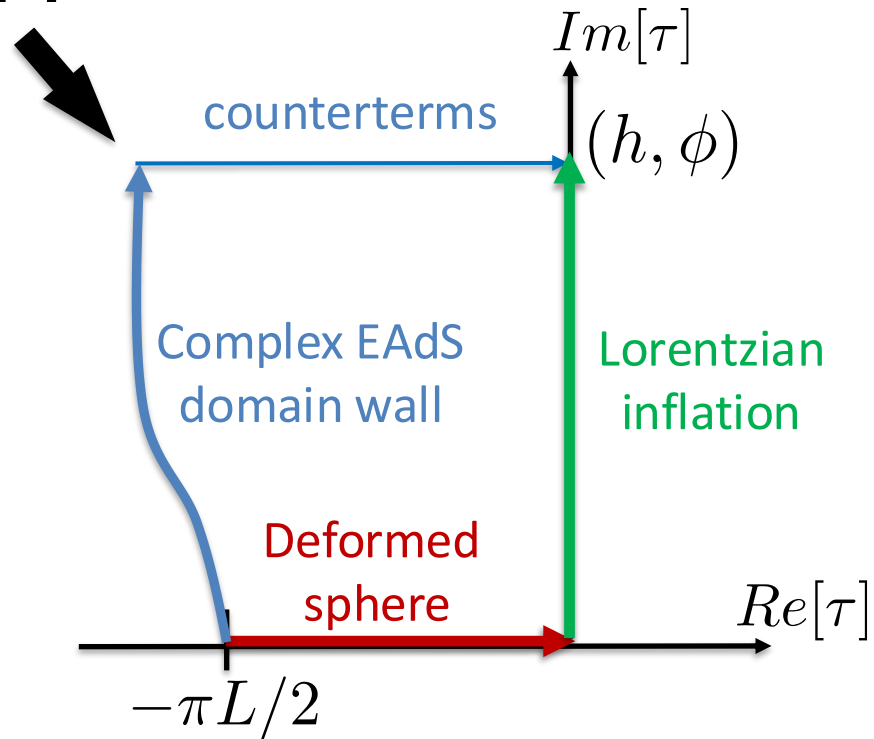


# Holographic no-boundary measure

$$\Psi_{HH} = \mathcal{A}_{sp} e^{iS}$$

$$\log \mathcal{A}_{sp} = I_{asEAdS}^{\text{reg}}$$

Dual QFT



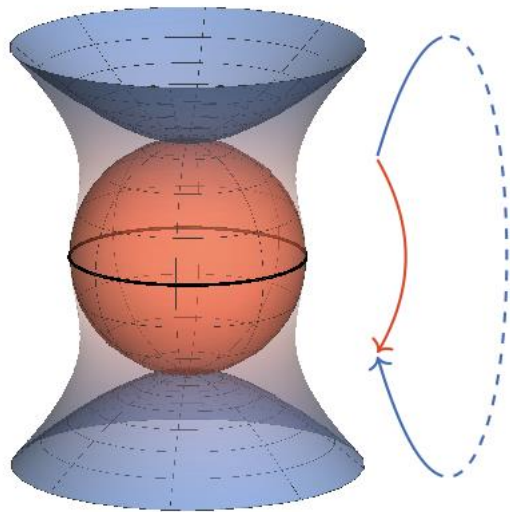
**Comment 1: Dual of NB condition?**  
You simply run out of d.o.f. ..

# Comment 2: measure, sphere, de Sitter entropy?

[Bobev, TH, Hong, Karlsson, Reys, '23]

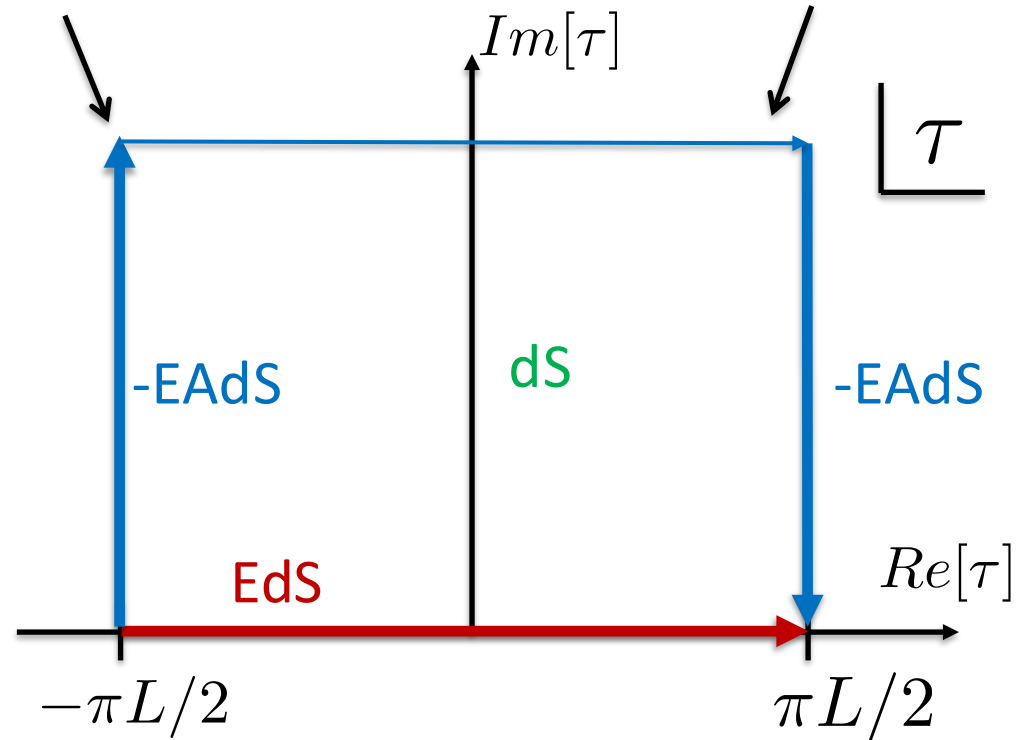
$$\Psi^* \Psi \sim e^{-I_{EdS}}$$

$$I_{EdS} = -2 I_{EAdS}^{\text{reg}}$$



**Dual QFT**

**Dual QFT**



## Comment 2: measure, sphere, de Sitter entropy?

Bobev, TH, Hong,  
Karlsson, Reys, 2023

$$S_{dS} = -I_{EdS} = 2I_{EAdS}^{\text{reg}}$$

- AdS/CFT :  $-I_{EAdS}^{\text{reg}} + \dots = \log Z_{S^3}^{\text{CFT}}$

- Conjecture:  $S_{dS} = -2 \log Z_{S^3}^{\text{CFT}}$

$$Z_{S^4}[\phi_i] \longleftrightarrow Z_{S^3}^{QFT}[c_i] Z_{S^3}^{QFT}[c_i]$$

## Comment 2: measure, sphere, de Sitter entropy?

Bobev, TH, Hong,  
Karlsson, Reys, 2023

$$EAdS_4 \times S^7 / \mathbb{Z}_k$$

$$S_{dS} = -I_{EdS} = 2I_{EAdS}^{\text{reg}} = -2 \log Z_{S^3}^{\text{ABJM}}$$

[Marino, Putrov; Fuji, Hirano, Moriyama; many others...]

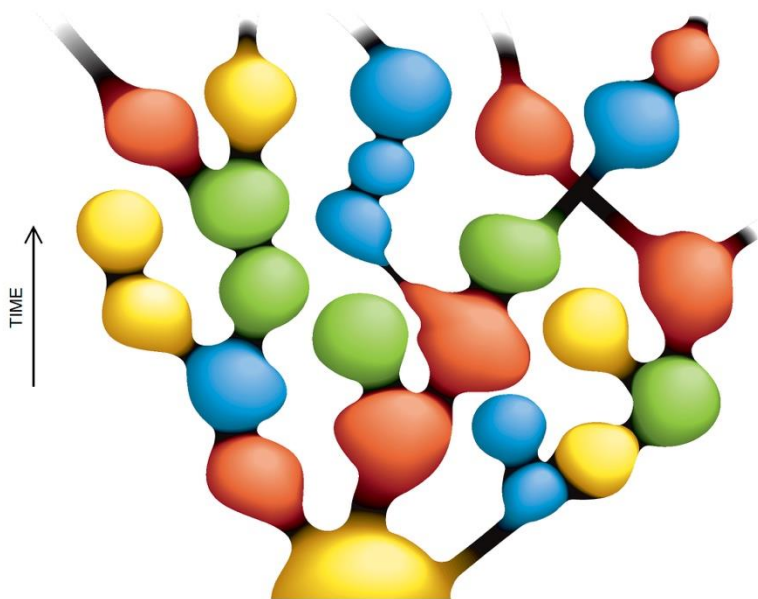
$$S_{dS} = \frac{2\pi\sqrt{2k}}{3} N^{3/2} - \frac{\pi(k^2 + 8)}{12\sqrt{2k}} N^{1/2} + \frac{1}{2} \log N + \mathcal{O}(N^0)$$

- Leading term: matches Gibbons-Hawking entropy
- Subleading term: higher-derivative terms in sugra

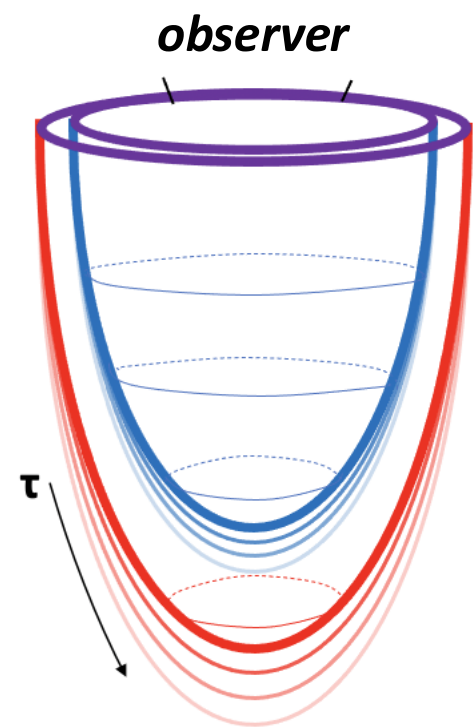
[Bobev, Charles, Hristov, Reys '21]

- Log correction: one-loop determinant in 11D Eucl SUGRA on  $-S^4 \times S^7 / \mathbb{Z}_k$

[Bhattacharyya, Grassi, Marino, Sen]



$\Psi$ , Observer  $\rightarrow$



Thank you

# de Sitter entropy: microscopics

- Consider 11d Euclidean SUGRA on  $-S^4 \times S^7 / \mathbb{Z}_k$
- One-loop determinants generate log corrections to the free energy
- Odd dimensions: only zero modes contribute
- Massless 11d fields: metric, gravitino and three-form
- Ghosts are important!
- Metric and gravitino have no zero mode because  $S^4$  is compact.
- Logarithmic correction due to a p-form:

$$\Delta F = \sum_j (-1)^j (\beta_{p-j} - j - 1) n_{\Delta_{p-j}}^0 \log L/l_P, \quad \beta_k = \frac{D - 2k}{2}$$

- $\rightarrow \Delta S_{dS} = 3 \log L/l_P \quad S_{dS} \stackrel{v}{=} -2 \log Z_{S^3}^{\text{ABJM}}$