# BRA-KET WORMHOLES IN TWO & FOUR DIMENSIONS

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Observers, wormholes & $\mathbb{C}$ saddles in cosmology

BASED ON 2408.08351 w/ GORBENKO, KAMES-KING

& 260X.XXXX w/ $\underbrace{\hphantom{GORBENKO, KAMES-KING}}$ + JANSSEN, VARRONE

# OUTLINE

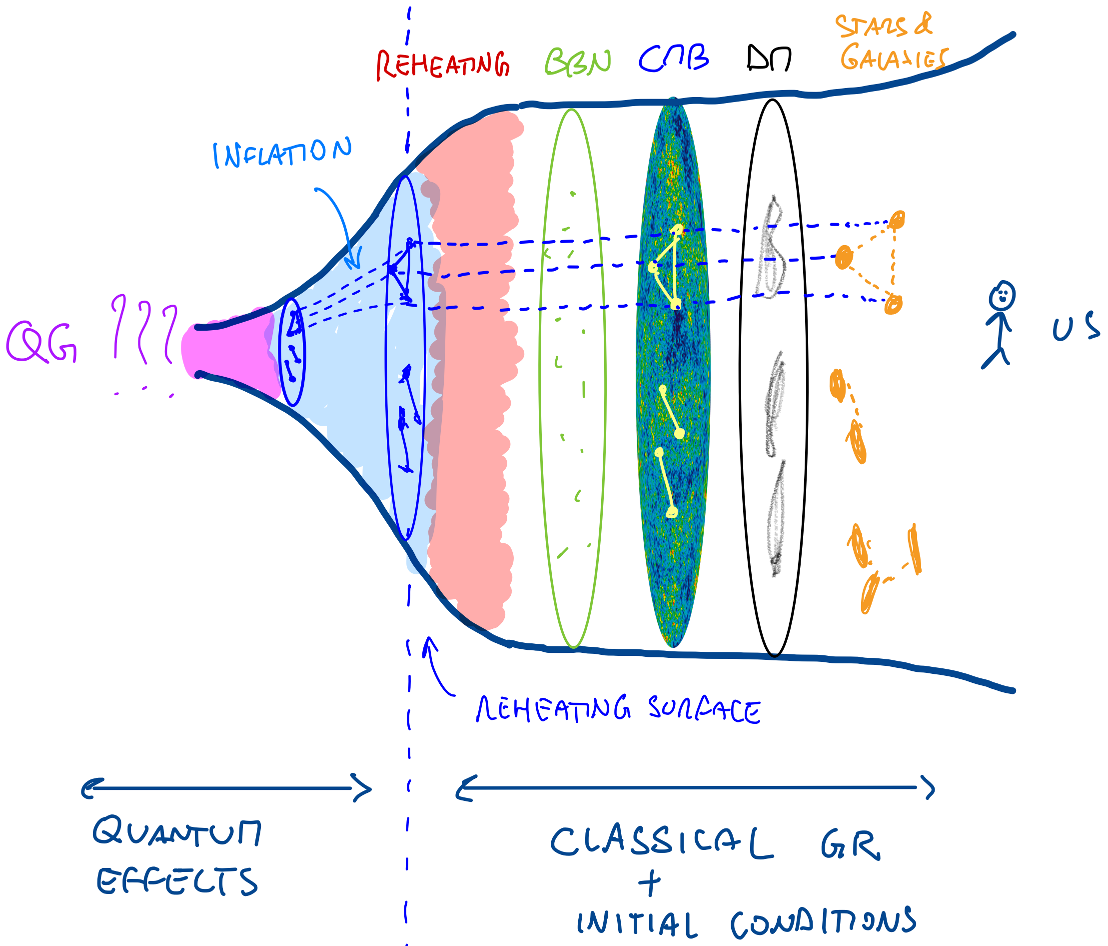
* What are Bra-Ket wormholes & why are they interesting

* Bra-ket wormholes in **two** dimensions

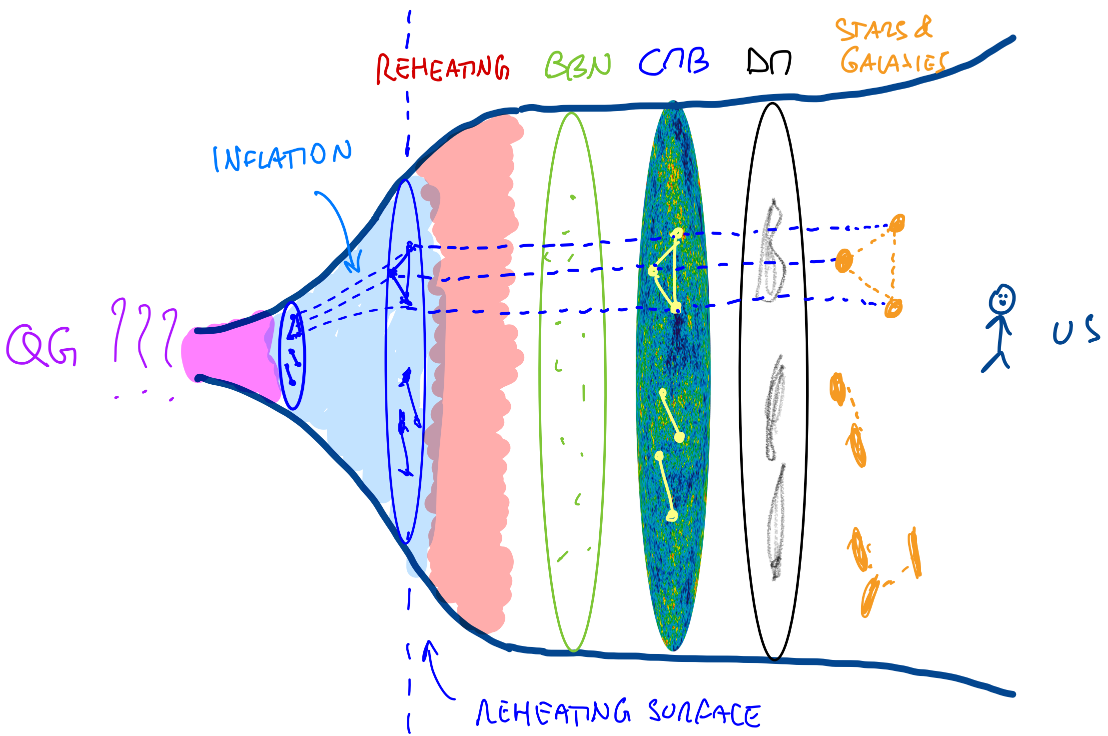
* Bra-ket wormholes in **four** dimensions

# QUANTUM EFFECTS IN COSMOLOGY





# QUANTUM EFFECTS IN COSMOLOGY




QUANTUM EFFECTS → Prob ( INITIAL CONDITIONS )

CLASSICAL GR
+
Prob ( INITIAL CONDITIONS )

OBSERVABLES:

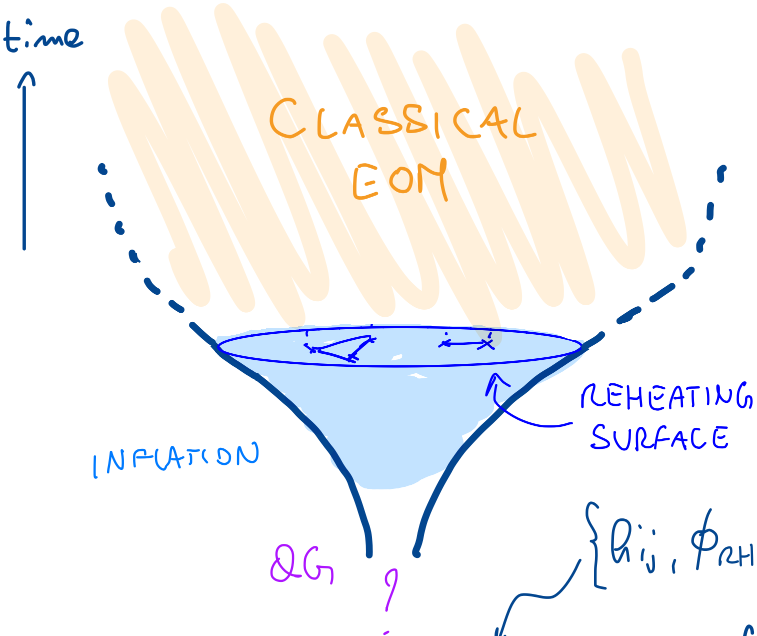
- $\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle$  $\zeta \sim \delta T^{CMB}$
- $N_e$  $\left( n_s - 1, \frac{\text{tensor}}{\text{scalar}} \right)$
- $\Omega_k$

# THE WAVE-FUNCTION OF THE UNIVERSE



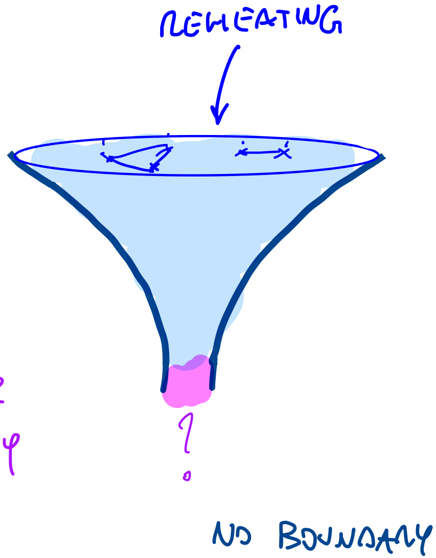
☑ Probability distr. @ reheating $= |\psi(\text{fields @ RH})|^2$

(but could also use other representations of QM: $\rho_\psi$, phase space QM ...)




$$= \sum$$

$\equiv$ geom. & topology




$\{h_{ij}, \phi_{RH}\}$

$$\Psi(x_{RH}) = \int_{\text{NO BOUNDARY}}^{x_{RH}} Dg\, Dx\; e^{iS[g,\phi]}$$

[Hartle, Hawking, '83]

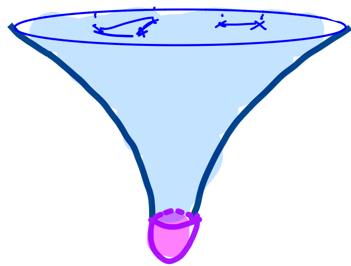
# THE WAVE-FUNCTION OF THE UNIVERSE



☑ Semiclassically

smooth homogeneous solutions

$$\Psi(\chi_{RH}) = \sum_{g_{cl}, x_{cl}} e^{i S_{cl}[g_{cl}, \phi_{cl}]} \int D\delta g, \delta\phi \; e^{i\delta S}$$




Prob ( homogeneous modes )

e.g. $N_{e\text{-folds}}$ :

Dominant saddle (HH): $N_e \to 0$

<u>disagrees</u> with observations

Prob ( short wavelength perturbations )

e.g. $\langle \delta_k \delta_{-k} \rangle \sim \langle \delta T_{CMB} \, \delta T_{CMB} \rangle$

Dominant saddle (HH): scale invariant pert.

<u>agrees</u> with observations

# BRA-KET WORMHOLES

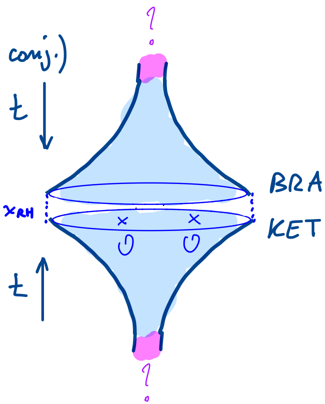


☒ Observables "connect" BRA & KET

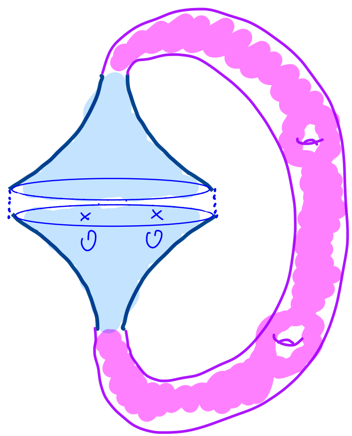
$$\langle \mathcal{O}\mathcal{O}\cdots\rangle = \int Dx_{RH}\, \underbrace{\psi^*(x_{RH})}_{\text{BRA}}\, \mathcal{O}\mathcal{O}\cdots \underbrace{\psi(x_{RH})}_{\text{KET}} = \int Dx_{RH}\, \rho(\overset{\text{BRA}}{x_{RH}}, \overset{\text{ket}}{x_{RH}})\, \mathcal{O}\mathcal{O}\cdots =$$

(complex conj.)

$$= \sum \int Dx_{RH}$$

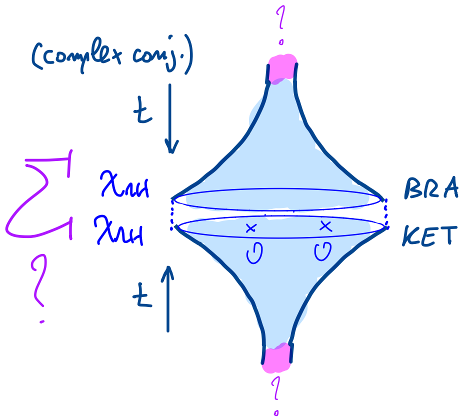



$$\supset \sum \int Dx_{RH}$$

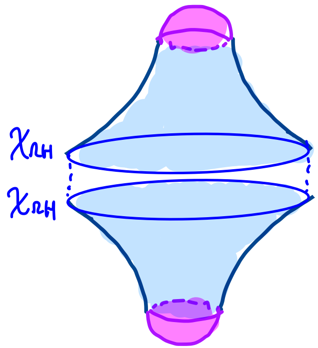


[Page '86] [Hawking '87] ... [Chen, Gorbenko, Maldacena '20]

# Bra-Ket Wormholes




ON-SHELL

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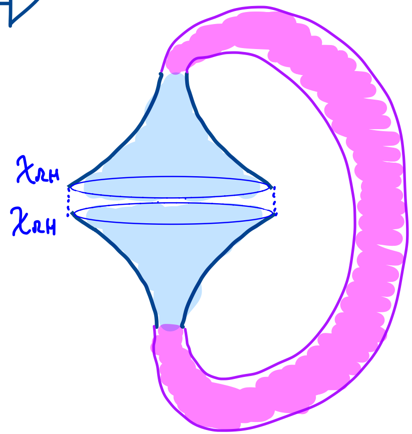
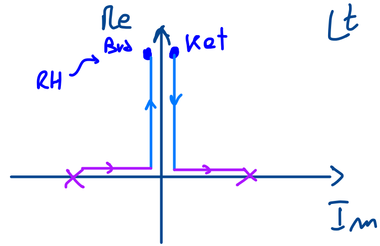


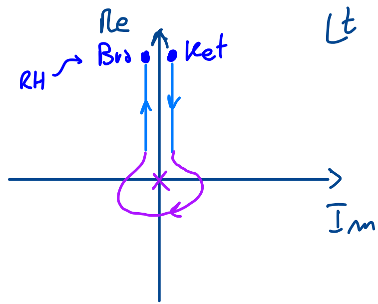

+ ....

## Hartle-Hawking state




$$ds^2 = -dt^2 + a(t)^2 d\Sigma_d^2$$

## Bra-Ket Wormhole




(similar to wormholes leading to ramp in SFF)

# BRA-KET WORMHOLES



Can we find classical solutions of this type? Yes (we show it in D=2, D=4)

What are their properties?

- Do they solve issues of HH?

In D=2 yes (but a different one). In D=4 we have a class of models that are promising

[Chen, Gorbenko, Maldacena; '20]

- Can they give the dominant contribution to observables?

In D=2 yes (large universes). In D=4

# THE 2D STORY (dS JT GRAVITY)



Bra-Ket wormholes $\exists$ as saddles of Wigner (phase space) distribution

$$W(L,P|\phi_b) \sim \int d\Delta\ell \; \rho(\ell_K,\ell_B|\phi_b)\, e^{iP\Delta\ell}$$

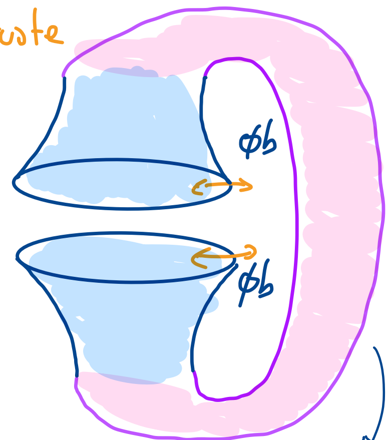
$\phi_b$: clock

$\rho$: time

fluctuate

$\ell_K = L + \Delta\ell$

$\ell_B = L - \Delta\ell$

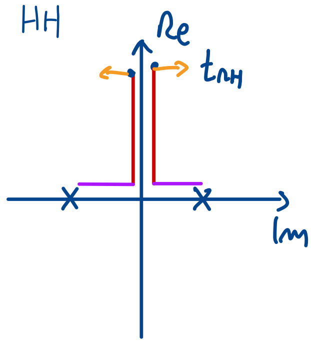


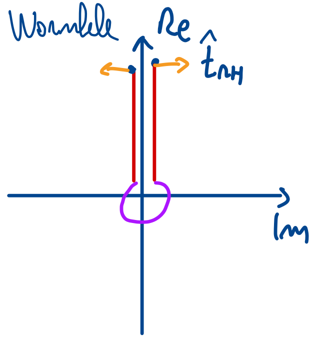

$L = \ell_K + \ell_B$ fixed

$P \sim$ conjugate variable to $\Delta\ell$

In QM $W(x,P|t)$ :

- $\langle \hat{O} \rangle \sim \int d(\text{phase space})\; W\, O_W$
- semiclass. $W \sim N\, \delta\left(P - P_{\text{classical}}(x)\right)$



# WHY WIGNER ?



☒ Natural in cosmology

$W$ = probability distribution for coord. & momenta!

initial conditions for classical hot Big Bang evolution.

☒ $\nexists$ wormhole solutions for density matrix [Chen, Gorbenko, Maldacena; '20]

↳ $\rho$ is singular (semiclassically)

☒ Wigner provides a regularization (~ microcanonical ensemble for double-cone in AdS) & has a better semiclassical limit

# Our Solution

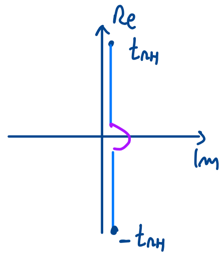
[AF, Gorbenko, Kames-King '24]

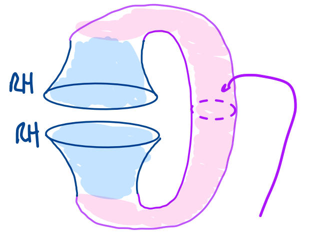


- We consider JT dS + INFLATON $\chi \longrightarrow$ clock

  four dim. phase space $L, \Phi$ & $P_L, P_\phi$

$$W(L, \phi, P_\phi, P_L \,|\, \chi b) \sim \delta(L\phi - P_\phi P_L)\, \mathcal{N}_{1\text{-loop}}$$




- Classically:

$$\Phi = \phi_r \cosh(t) \qquad L = b \sinh(t)$$

$$P_\Phi = b \cosh(t) \qquad P_L = \phi_r \sinh(t) \implies L\phi - P_\phi P_L = 0$$

$\hookrightarrow$ $W \sim \delta(\text{classical trajectories})$ !

# FEATURES OF OUR SOLUTION



☑ Dominates @ large $L$
@ large Universe

$$W_{HH} \sim \frac{1}{L^3} e^{S_{dS}}$$

Adding a CFT

$$W_{\text{wormhole}} \sim \frac{L}{\Phi^2} e^{c\, b_{cl} H_*}$$

☑ Solves an issue of HH in $D=2$ (violation of SSA)

[Chen, Gorbenko, Maldacena; '20]

☑ NON NORMALIZABLE due to $L \to \infty$ ⇝ $\rho$ doesn't exist
(could be bounded by higher topologies)

☑ Short wavelength matter perturbations are scale invariant
Longer ones (wavelength ≳ throat size)
are thermal

# Four-Dimensional Bra-Ket Wormholes

Moving to 4D: see if we can resolve the HH issue.

WIP 260X.XXXX

uplift of JT dS

- ☒ Different boundary topology: e.g. $S^3$, $S^2 \times S^1$, $S^1 \times S^1 \times S^1$

  + open Universes

- ☒ Add spatial curvature $\Omega_K \gtreqless 0$

- ☒ Anisotropic wormholes

- ☒ Many inequivalent contours in $\mathbb{C}$ time $\leftrightarrow$ many different saddles

  each with new properties

# ANALYZING MATTER ON GENERIC SOLUTIONS



Matter should behave nicely on these backgrounds, to yield perturbations close to what we observe. Introduces two criteria:

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Matter should behave nicely on these backgrounds, to yield perturbations close to what we observe. Introduces two criteria:

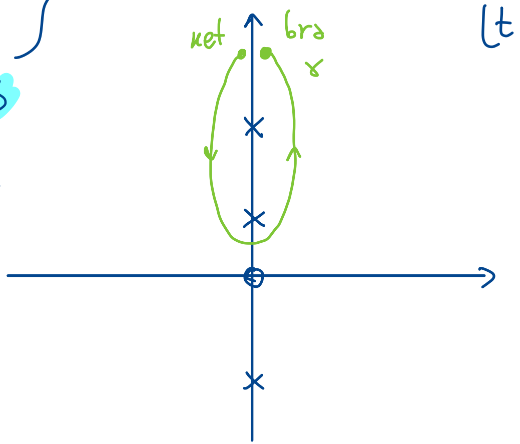
- Place a CFT on the background
  → thermal state @ temp $\beta^{-1}$

$$\beta = \int_{ket}^{bra} dy = \int_{\gamma} \frac{dt}{f(t)}$$

REQUIRE $\begin{cases} \beta \in \mathbb{R} \\ \beta > 0 \end{cases}$

e.g.: Schwarzschild dS ($S^1 \times S^2$) — uplift of JT dS

$$ds^2 = -\frac{dt^2}{f} + f\,d\omega^2 + R^2 d\Omega_2^2$$



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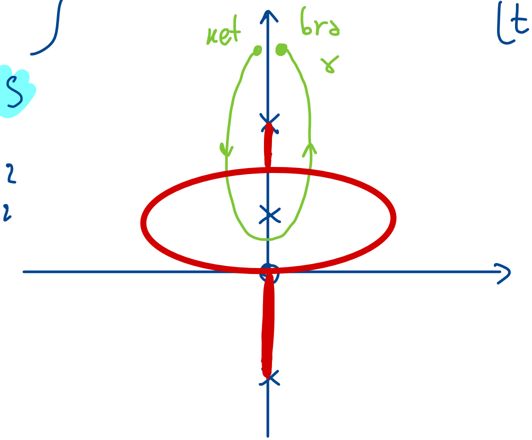


<u>REQUIRE</u> $\begin{cases} \beta \in \mathbb{R} \\ \beta > 0 \end{cases}$

- Consider a CFT e.g. c.c. scalar $+ \lambda \chi^4$

$$\int D\chi \; e^{\,i\int \sqrt{-g(t)}\left(g^{\mu\nu}(t)\partial_\mu \chi \partial_\nu \chi + \lambda \chi^4\right)}$$

Bounded @ any $t \in \gamma$ ⇒ No ghost instability

↳ NO GO's (Conformal KSW)

Note: generic matter is this + a relevant deformation, e.g. a mass term. negative $m^2$ is <u>NOT</u> an instability.

# AN OVERVIEW OF SOME SETUPS WE STUDIED



- Pure gravity $\Lambda > 0$

  sols w/ boundary : $S^3$ , $S^1 \times S^2$ , anisotropic $S^1 \times S^1 \times S^1$

  uplift of JT dS

  (Kasner Universes)

# An Overview of Some Setups we Studied



- Pure gravity $\Lambda > 0$
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  NOT ALLOWED
  uplift of JT dS
  (Kasner Universes)

- Pure gravity $\Lambda > 0$ with hyperbolic boundary $H_3$.

  $\exists$ a selection with $\beta \sim H_*^{-1}$ & OK with conformal KSW
  however : $S = 0 \Rightarrow \rho \sim 1$

  $\ll$ Hartle-Hawking which has $\rho_{HH} \approx \exp\left\{ \frac{M_P^2}{H_*^2} \right\}$

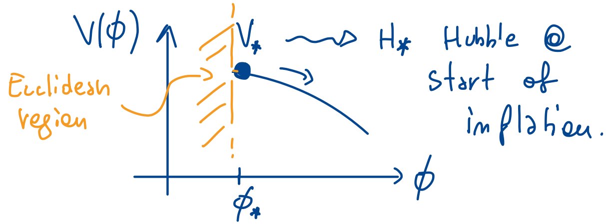
# A CLASS OF PROMISING MODELS



▨ Isotropic $S^1\times S^1\times S^1$ spatial topology + matter :

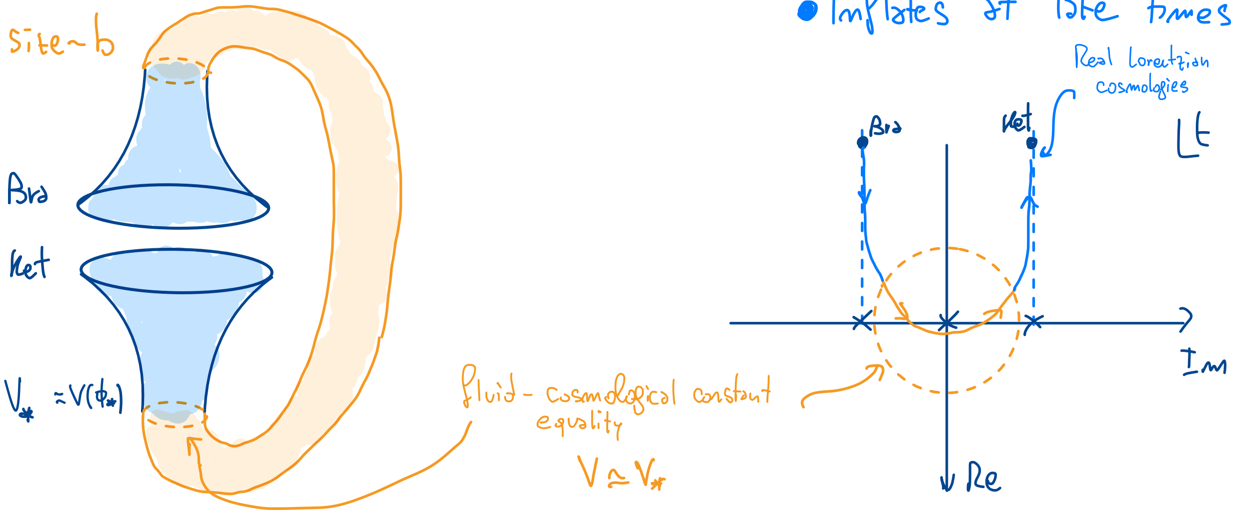
Gravity + Perfect fluid $P=\omega\rho$ + Slowly rolling inflaton $\phi$

microscopic field $\sigma$




- Controls the $\mathbb{C}$ throat (e.g. the size $\underline{\underline{b}}$)

- Inflates at late times



# A CLASS OF PROMISING MODELS



- First, consider RADIATION $\left(w = \frac{1}{3}\right)$, $\sigma$ is conf. coupled scalar
- They are similar to the JT wormhole : $\nexists$ density matrix

  $\exists$ Wigner

  clock      from Lorentzian region

$$W_{\text{wormhole}}(P_\phi, \Phi, P_\chi, \chi \,|\, L) \sim \delta\left(P_\phi - P_\phi^{cl}\right) \exp\left(I_{\text{throat}}\right)$$

from Euclidean throat

$$I_{\text{throat}} \simeq - M_p^2 \, H_* \, b^3$$

- These wormholes are different than the ones $\rho_{NB} = \mathbb{1}$
- $H_* \rightarrow N_{e\text{-folds}}$     $b^3 \rightarrow$ volume of Universe & depend on $P_\chi, \Phi, \ldots$

$$W_{\text{wormhole}} \ll W_{HH} \sim \exp\left\{\frac{M_p^2}{H_*^2}\right\}$$

However, ....

# A Class of Promising Models



For generic $\omega$ we have

$$I_{throat} \approx \pm_{(\omega)} M_p^2 H_* \times b^3$$

As in previous model, we expect:

$$W_{wormhole}(\Phi, P_\phi, x, P_x | L) \sim \delta(P_\phi - P_\phi^{cl}) \exp(I_{throat})$$

- $H(V_0) \to N_{e\text{-folds}}$ $\quad b^3 \to$ Volume of the universe
- The sign depends on $\omega$: if positive $\Rightarrow$ Universe large & $N_e$ large
- Moreover, $\quad W_{wormhole} \gg W_{HH} \approx \exp\left(\frac{M_p^2}{H_*^2}\right)$

Wormhole dominates over HH for large Universes

# CONCLUSION & OUTLOOK

## LESSONS FROM THE 2D TOY MODEL

- ↳ Bra-ket wormholes exist as saddles for Wigner distribution
- ↳ Are semiclassical & support stable matter
- ↳ Dominate over HH & solve its 2D issue
- ↳ Not normalizable in full phase space : $L \to \infty$ higher topologies !

## THE MORE REALISTIC 4D CASE

- ↳ Bra-ket wormholes exist as saddles for Wigner distribution
- ↳ Are semiclassical & support stable matter
- ↳ In a promising model they can potentially dominate over HH & solve the $N_{e\text{-fold}} \to 0$ issue
- ↳ Not normalizable in full phase space

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Thanks!

Backup

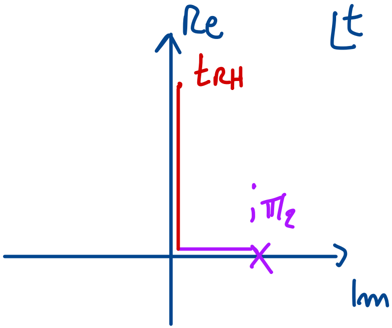
# THE HARTLE-HAWKING STATE

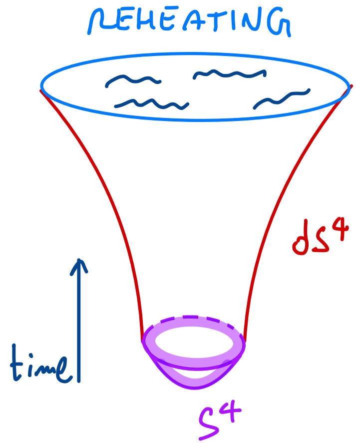
Semiclassically

$$\Psi_{sc} \simeq e^{i S_{cl}[g_{cl}^{HH}, \chi_{cl}^{HH}]} \int_{BD} \mathcal{D}\chi \; e^{i S[g_{cl}^{HH}, \chi]}$$

$dS_4$

$$ds^2 = -dt^2 + \cosh^2 t \, d\Omega_3^2$$




Bunch-Davies vacuum

$\Rightarrow$ SHORT WAVELENGTH CURVATURE PERT

$$\langle \zeta_k \, \zeta_{-k} \rangle \sim \frac{1}{|k|^3}$$

AGREES w/ OBSERVATIONS

# OUR SOLUTION IN D=2

- ☒ We use an infleton to fix time reparametrizations

- ☒ Phase space is four-dimensional $(L, P_L, \Phi, P_\Phi)$

  $\Phi \to$ dilaton

Define $U = \frac{P_L}{\Phi}$ $\quad V = \frac{P_\Phi}{L}$

- $U, V$ locally observable

  $\sim$ # e-folds $\equiv N_e$

- $L, \Phi$ overall size of 2D Universe & $S^2$

$$\boxed{W(L, \Phi, U, V) \sim \mathcal{N}_{\text{1-loop}}(L, \Phi)\, \delta(U - U_{cl}(V))}$$

indeed semiclassical!

# INTERPRETATION

$W \sim$ probability distr. for gravitational zero modes

$$\langle \hat{O} \rangle \sim \left( \int dL \int d\Phi\, \Phi^2 \right) \int \underbrace{du}\, \underbrace{dv}\, (1+\cdots)\, \underbrace{\delta(u - u_{cl}(v))}\, O^{loc}(u,v)$$

↳ integral over age of the Universe

$V \leftrightarrow N_e$

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$$\langle \hat{\mathcal{O}} \rangle \sim \int dL \int d\Phi\, \Phi^2 \int du\, dv\, (1+\cdots)\, \delta(u-u_{cl}(v))\, \mathcal{O}^{loc}(u,v)$$

$$\sim \int dN_e\, \underbrace{e^{-2N_e}}\, \mathcal{O}_{cl}(N_e)$$

- Still pushes for $N_e \to 0$
- Much milder than HH in 4D $\left( W_{HH} \approx e^{-2N_e}\, e^{2S_0} \right)$

analogous to $e^{1/V}$, not there in wormhole

# COMPARISON w/ HARTLE HAWKING



We can compute Wigner for Hartle-Hawking state :

$$\frac{W^{\text{WORMHOLE}}}{W^{HH}} \simeq \frac{L^4}{\Phi^2} e^{-2\phi_0}$$

WORMHOLE DOMINATES FOR LARGE UNIVERSE

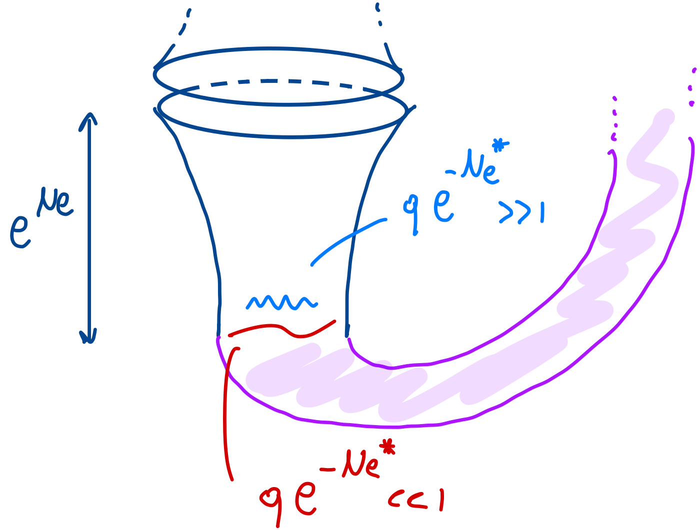
# MATTER CORRELATORS & POWER SPECTRUM



Consider a massless scalar $\chi$ (& project $N_e$ (or V) to $N_e^*$ large)

$$\langle \chi_q \, \chi_{-q} \rangle_{N_e^*} \, dq =$$

physical momenta




- $\frac{1}{4\pi} \frac{dq}{q}$ scale invariant

- $\frac{1}{2\pi^2} \frac{dq}{q^2}$ thermal $\beta \sim e^{-N_e^*}$