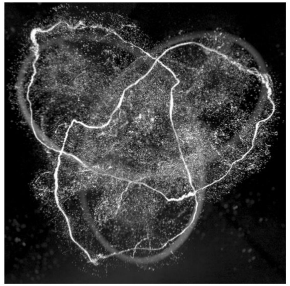
Out-of-time-ordered Transport

Luca Delacrétaz | U Chicago

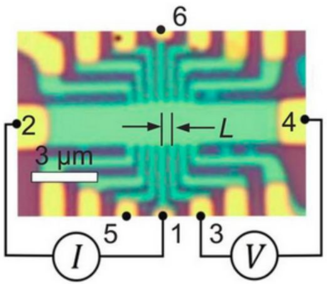
EPFL – Effective Field Theories for Hydrodynamics
16 Dec 2025

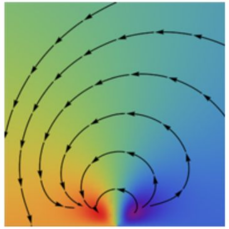
Hydro is everywhere

Fluids

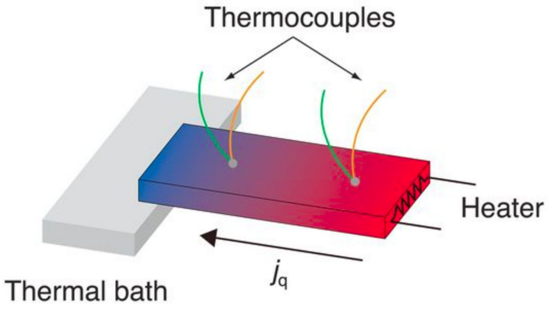


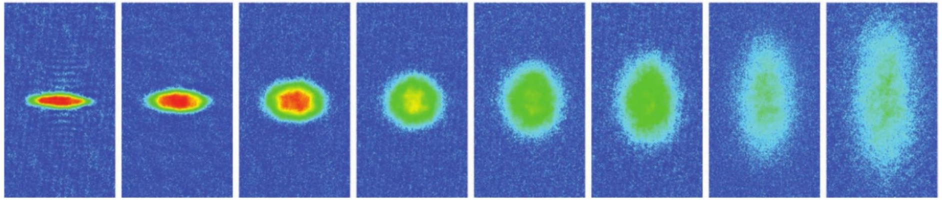
Electron hydro



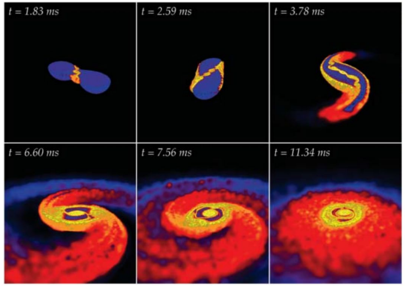


Heat diffusion



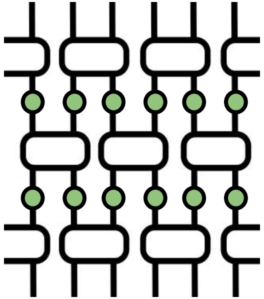


Cold atoms

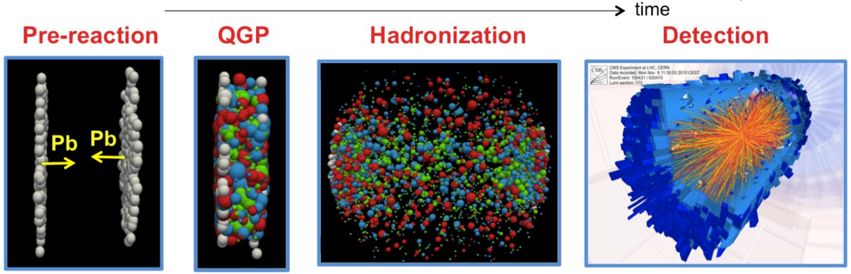


Neutron stars

Floquet, RUC, …



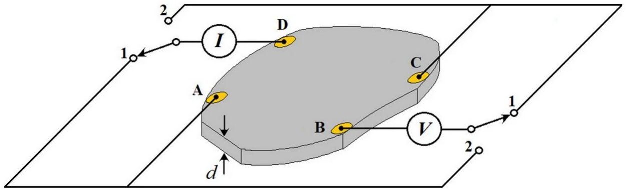
QGP



Hydrodynamic signatures in quantum observables

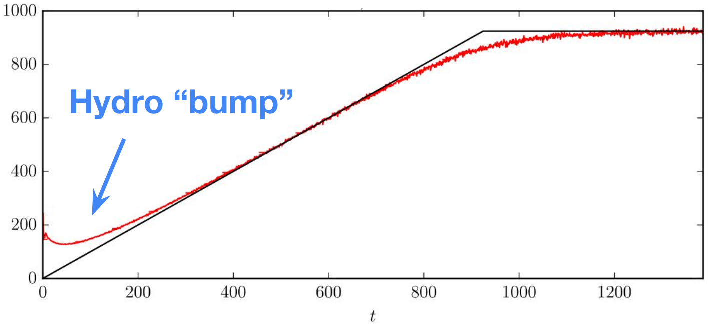
Transport, regular correlators

Landau Lifshitz, Kadanoff Martin '63 …



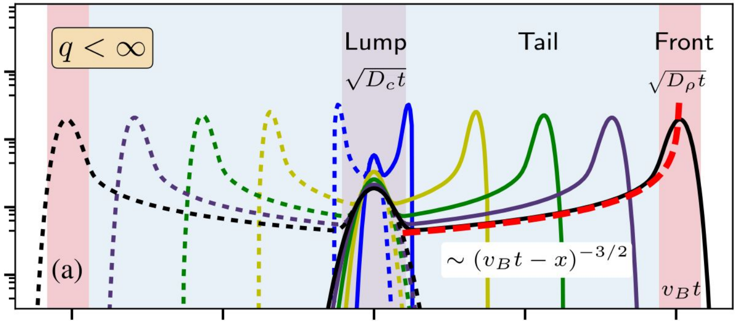
Spectral form factor

Friedman Chan DeLuca Chalker '19
Swingle Winer '20



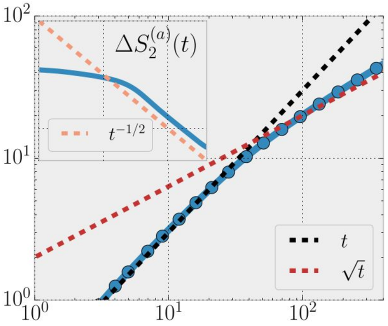
OTOCs

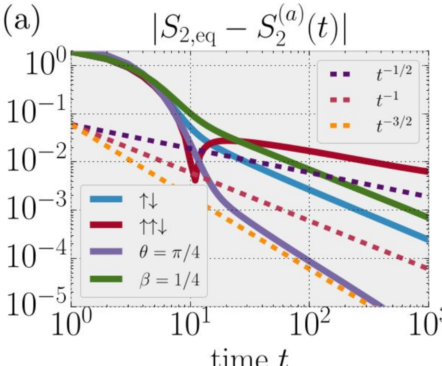
Rakovszky Pollmann Keyserlingk '18,
Khemani Vishwanath Huse '18, …



Entanglement dynamics

Rakovszky Pollmann Keyserlingk '19
Huang '19, Znidaric '19, …





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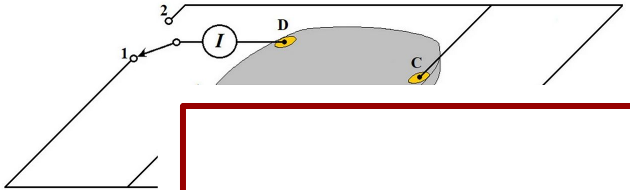

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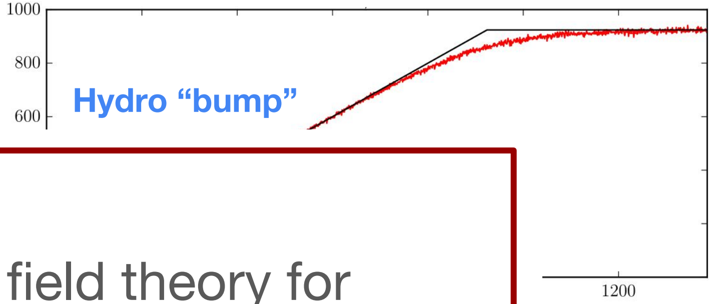
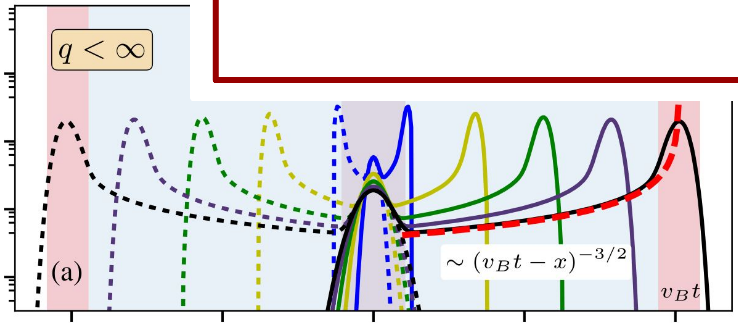


“Beyond MSR” effective field theory for

fluctuating hydrodynamics?

OTOCs



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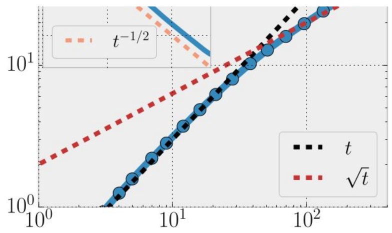
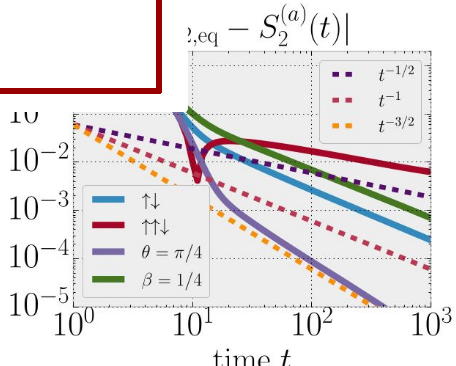
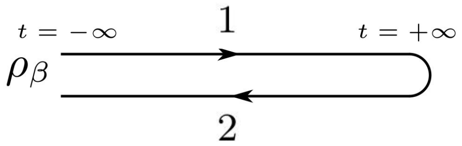




Schwinger-Keldysh EFTs for hydro



Hydrodynamics as an EFT on a Keldysh contour

Grozdanov Polonyi '14, Haehl Loganayagam Rangamani '15, Crossley Glorioso Liu '15, Jensen Pinzani-Fokeeva Yarom '17

Hydro = Nambu-Goldstone modes of strong to weak SSB

Ogunnaike Feldmeier Lee '23, Akyuz Goon Penco '23, Gu Wang Wang '24, Huang Qi Zhang Lucas '24, Firat Gomes Nardi Penco Rattazzi '25

Most applications (many exciting) could have been obtained from MSR

Martin Siggia Rose '73 De Dominicis '76 Janssen '76

Today: Fluctuating hydro for intrinsically quantum observables

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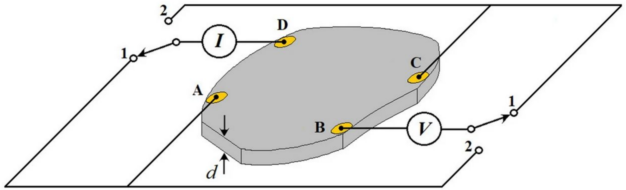

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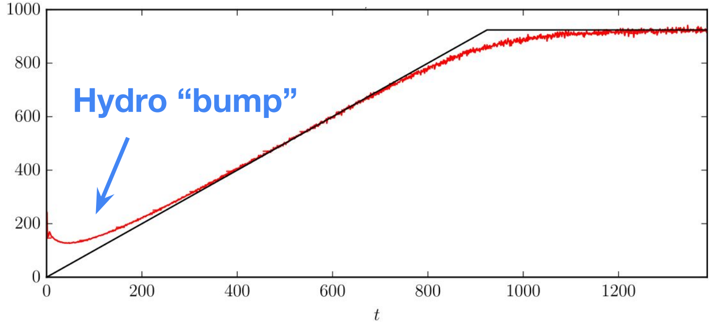

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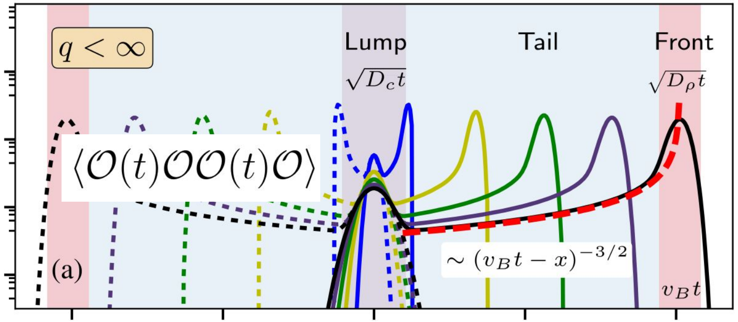

Entanglement dynamics

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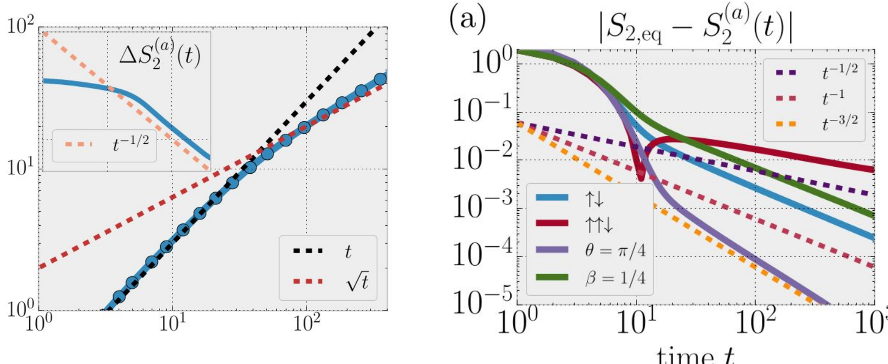

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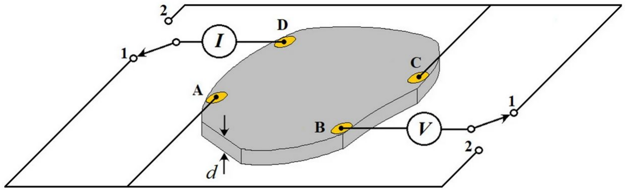

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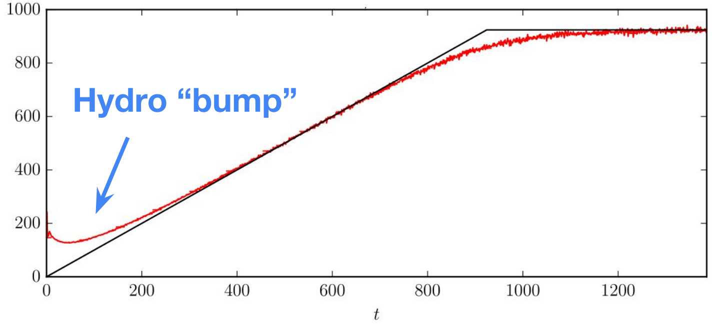

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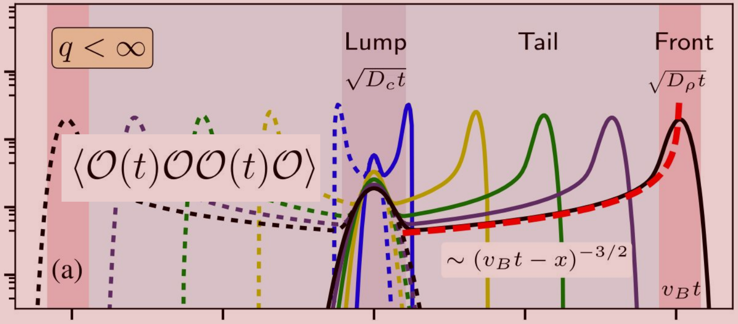

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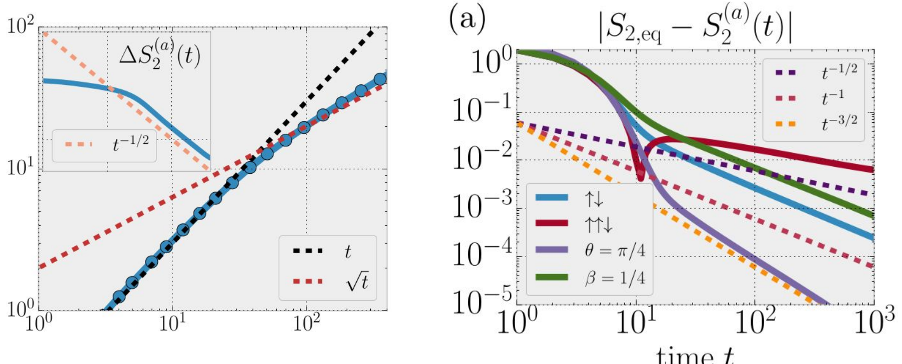
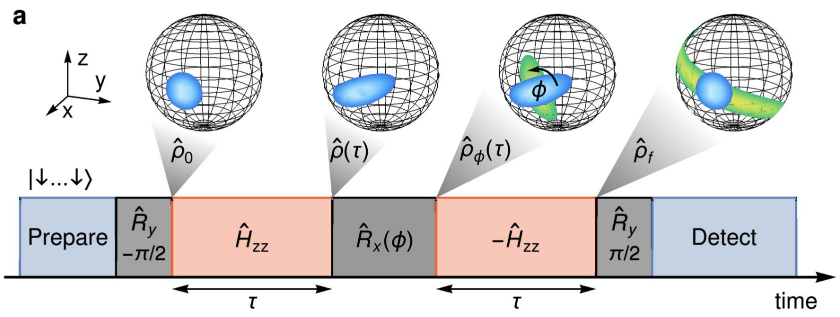
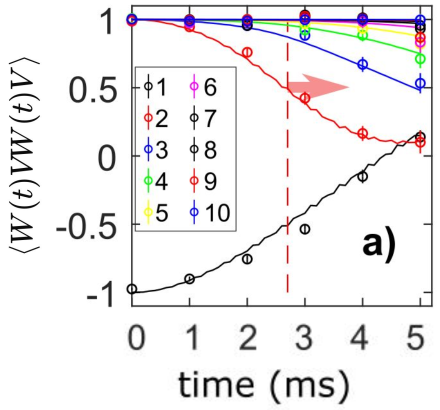


OTOCs in experiments

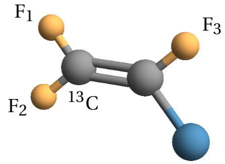
Trapped ions
Gärttner Bohnet et al ‘17, Landsman et al ‘18, Joshi et al ‘20

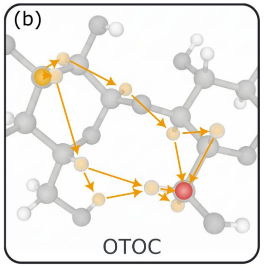


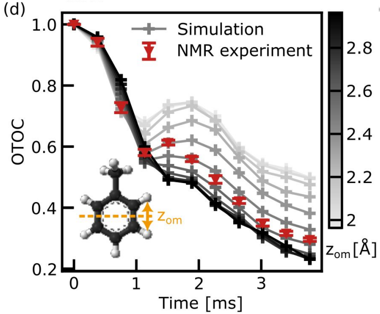
$$\overline{\langle W(t)\rangle_U \langle V\rangle_U} \propto \langle W(t)VW(t)V\rangle$$



NMR
Li et al ‘17, Wei et al ‘17, Nie et al ‘19, …, Zhang et al ‘25



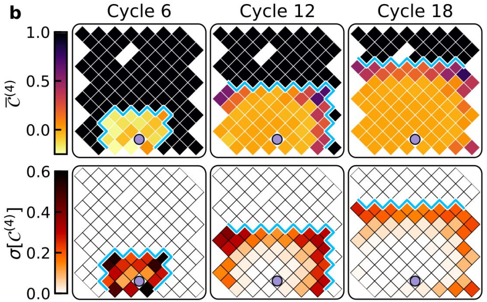




Superconducting quantum circuits

Google quantum AI, ‘21-‘25,

Braumüller et al, ‘21



Results

arXiv:2512.11198 with

Ruchira Mishra, Jiaozi Wang, Silvia Pappalardi



Earlier studies focused on OTOCs at coincident points

$$\langle \mathcal{O}(t)\mathcal{O}\mathcal{O}(t)\mathcal{O}\rangle \sim 1/t^{1/2}$$

Rakovszky Pollmann Keyserlingk ‘18, Khemani Vishwanath Huse ‘18, …

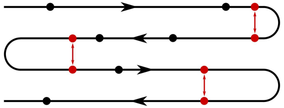
“Long-distance” OTOCs scale differently

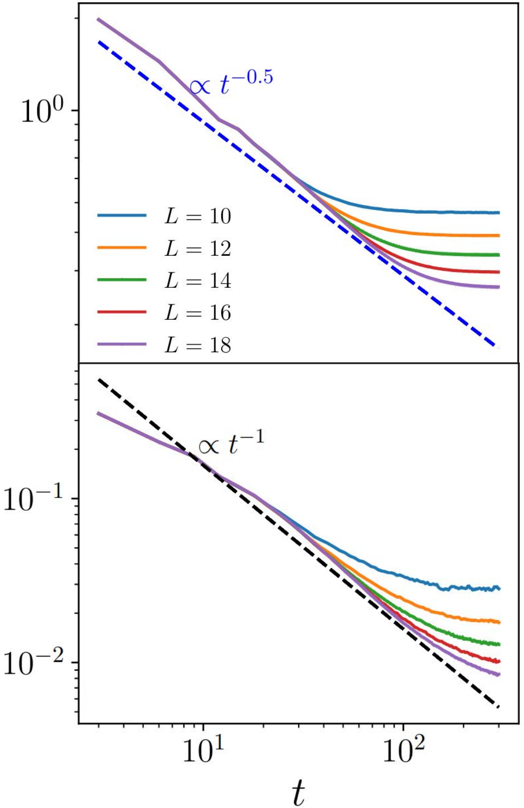
$$\langle \mathcal{O}(3t)\mathcal{O}(t)\mathcal{O}(2t)\mathcal{O}\rangle \sim 1/t$$
$$\langle \mathcal{O}(3t)\mathcal{O}(t)\mathcal{O}(2t)\mathcal{O}\rangle_c \sim 1/t^{3/2}$$

New OTO-transport parameters: $\lim_{\omega,k\to 0}\langle [\mathcal{O},\mathcal{O}]^2\rangle = \lambda$

Adds to conventional transport data $\chi,\ \sigma_{\mathrm{dc}},\ \ldots$

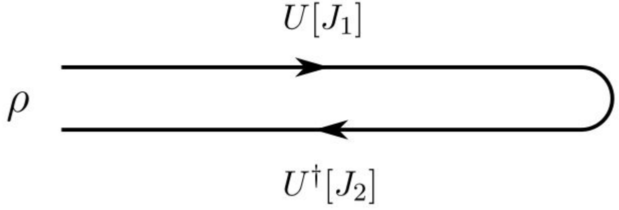
Generalization of “strong-to-weak SSB”





Constructing the EFT

Keldysh contour for real time dynamics



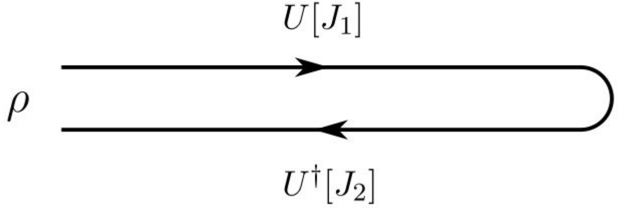
To study $\mathrm{Tr}\left(\rho\mathcal{O}(t_1)\mathcal{O}(t_2)\cdots\right)$ must time-evolve mixed state $\rho(t) = e^{-iHt}\rho e^{iHt}$

Couple to sources: $U(t_f, t_i)[J] = Te^{-i\int_{t_i}^{t_f} H(t)+J(t)\mathcal{O}(t)}$

Generating functional:

$$Z[J_1, J_2] \equiv \mathrm{Tr}\left(U(\infty, -\infty)[J_1]\rho U^\dagger(\infty, -\infty)[J_2]\right) = \int_{\mathrm{BC}} D\psi_1 D\psi_2\, e^{iS_1 - iS_2}$$

Schwinger-Keldysh contour

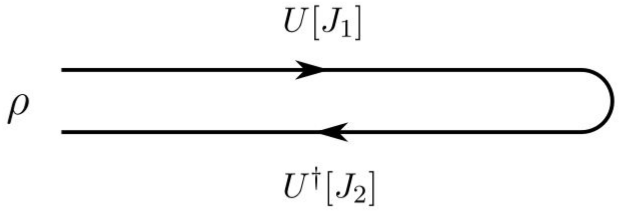


Some general conditions:

$$Z[J_1, J_2] \equiv \mathrm{Tr}\left(U[J_1]\rho U^\dagger[J_2]\right)$$

1. $Z[J, J] = 1$ — "collapse" rule
2. $Z[J_1, J_2]^* = Z[J_2, J_1]$ — unitarity
3. $Z[J_1, J_2] = Z[J_1(-t + i\beta), J_2(-t)]$ — KMS (+time reversal)

Schwinger-Keldysh contour



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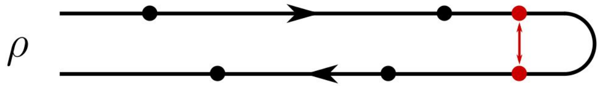
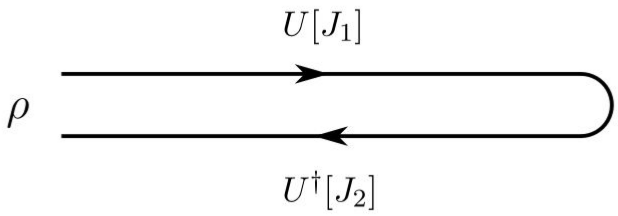
More refined version of 1. : latest operator can be inserted on either leg

$$\langle \mathcal{O}_{i_1}(t_1) \cdots \mathcal{O}_{\mathbf{1}}(t_n)\rangle = \langle \mathcal{O}_{i_1}(t_1) \cdots \mathcal{O}_{\mathbf{2}}(t_n)\rangle$$



(trace cyclicity)

Symmetries of mixed state evolution



Buca Prosen '12

Natural notion of doubled symmetries on mixed states

$$\rho \to e^{-i\alpha_1 Q} \rho e^{i\alpha_2 Q} \qquad U(1) \times U(1)$$

Useful in the context of *open* dynamics, where Lindblad evolution can break either symmetry

$$\partial_t \rho = \mathcal{L}\rho \equiv -i[H,\rho] + \sum \left(2L_i \rho L_i^\dagger - L_i^\dagger L_i \rho - \rho L_i^\dagger L_i\right)$$

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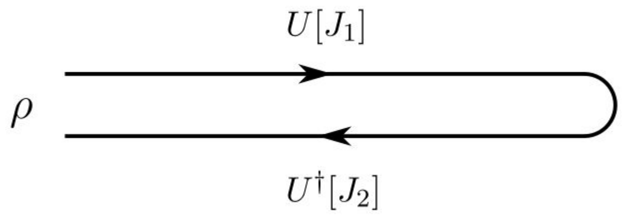

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If L_i is charged, dynamics only has "weak" symmetry $U(1)_{\text{diag}}$

Why care for closed systems? Thermal states have "strong to weak" SSB

$$\text{Tr}\left(\rho \psi^\dagger \psi\right) = \langle \psi_2^\dagger \psi_1 \rangle \neq 0 \qquad U(1) \times U(1) \to U(1)_{\text{diag}}$$

Ogunnaike Feldmeier Lee '23, Akyuz Goon Penco '23, Lessa Ma Zhang Bi Cheng Wang '24, Gu Wang Wang '24, Huang Qi Zhang Lucas '24, …

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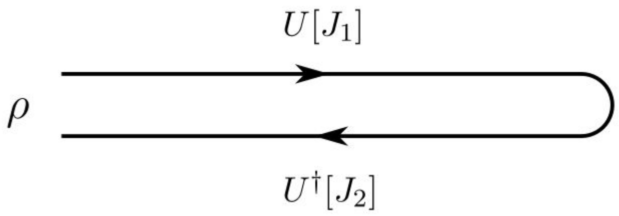

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Hydrodynamics

"Goldstone modes for strong to weak SSB"

Useful in the ... ak either symmetry

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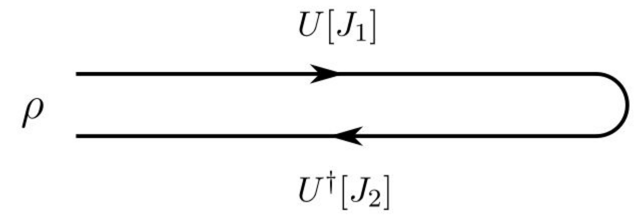

Natural notion of doubled symmetries on mixed states

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Useful in the ... ak either
symmetry

If L_i is char

Why care fo

Hydrodynamics

"Goldstone modes for strong to weak SSB"

Unifying description for open systems [review: Sieberer Buchhold Diehl '16]

Builds on previous SK EFTs Haehl Loganayagam Rangamani '15, Crossley Glorioso Liu '15, Jensen Marjieh Pinzani-Fokeeva Yarom '18

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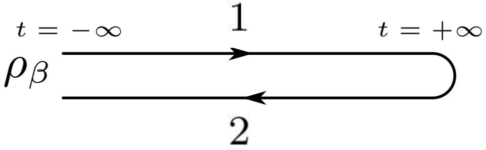
EFT for fluctuating hydrodynamics



If $G_1 \times G_2$ were completely broken, late time EFT would contain two Goldstones:

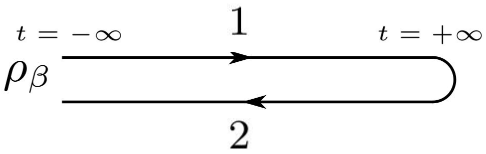
$$\phi_{1,2} \to \phi_{1,2} + c_{1,2}$$

Convenient to work with Keldysh rotated fields $\phi_a = \phi_1 - \phi_2$ and $\phi_r = \frac{1}{2}(\phi_1 + \phi_2)$

Effective action describes a superfluid (ordered phase), with $\omega \sim q$

$$S = \int dt d^d x \left[\dot{\phi}_a^2 + (\nabla \phi_a)^2 + \dot{\phi}_a \dot{\phi}_r + \nabla \phi_a \cdot \nabla \phi_r + \cdots \right]$$

EFT for fluctuating hydrodynamics



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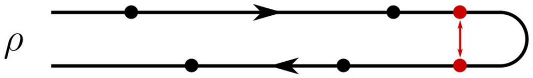
No ϕ_r^2 term, to guarantee $\langle \phi_a \phi_a \rangle = 0$

$$\langle \mathcal{O}_{i_1}(t_1) \cdots \mathcal{O}_{\mathbf{a}}(t_n) \rangle = 0$$



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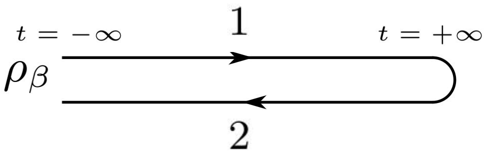

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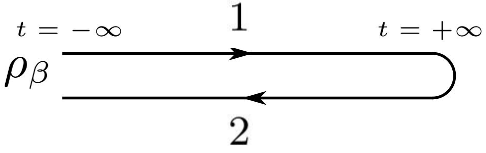
$$S = \int dt d^d x \left[\dot{\phi}_a^2 + (\nabla \phi_a)^2 + \dot{\phi}_a \dot{\phi}_r + \nabla \phi_a \cdot \nabla \phi_r + \cdots \right]$$

EFT for fluctuating hydrodynamics



If $G_1 \times G_2$ were completely broken, late time EFT would contain two Goldstones:

$$\phi_{1,2} \to \phi_{1,2} + c_{1,2}$$

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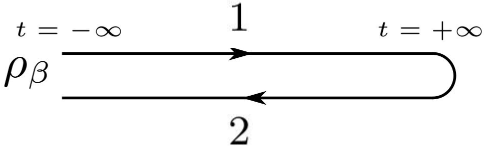
KMS symmetry:

$$Z[J_1, J_2] = Z[J_1(-t + i\beta), J_2(-t)]$$

$$\begin{cases} \phi_a \to -(\phi_a + i\beta\dot{\phi}_r) + \cdots \\ \phi_r \to -(\phi_r + \frac{1}{4} i\beta\dot{\phi}_a) + \cdots \end{cases}$$

EFT for fluctuating hydrodynamics



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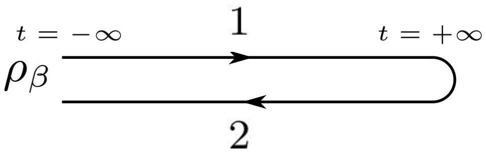
When instead $G_1 \times G_2 \to G_{\text{diag}}$, EFT contains 1 Goldstone, and 1 Keldysh “friend”

$$\phi_a \to \phi_a + c\,, \qquad \mu_r\,.$$

⇒ diffusion !

$$S = \int dtd^dx \;\left[\chi\dot\phi_a\mu_r + \sigma\nabla^2\phi_a\mu_r + iT\sigma(\nabla\phi_a)^2 + \cdots\right] \qquad \omega \sim q^2$$

EFT for fluctuating hydrodynamics



KMS multiplet:

$$\begin{cases} \phi_a \to -(\phi_a + i\beta\mu_r) + \cdots \\ \mu_r \to \mu_r + \frac{1}{4} i\beta\ddot{\phi}_a + \cdots \end{cases}$$

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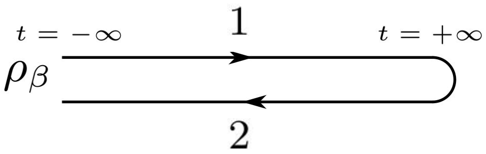
$$S = \int dt d^d x \left[\chi\dot{\phi}_a\mu_r + \sigma\nabla^2\phi_a\mu_r + iT\sigma(\nabla\phi_a)^2 + \cdots \right] \qquad \omega \sim q^2$$

KMS symmetry

EFT for fluctuating hydrodynamics



One can identify the hydrodynamic density as the field conjugate to ϕ_a :

$$n \equiv \frac{\delta S}{\delta \dot{\phi}_a} = \chi \mu_r + \cdots$$

When instead $G_1 \times G_2 \to G_{\mathrm{diag}}$, EFT contains 1 Goldstone, and 1 Keldysh “friend”

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$\Rightarrow$ diffusion !

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$$\omega \sim q^2$$

EFT for fluctuating hydrodynamics $Z[A_\mu^1, A_\mu^2] \simeq \int DnD\phi_a \, e^{i\int dtd^dx\,\mathcal{L}}$

Using symmetries and properties of the generating functional, the EFT takes the form

$$\mathcal{L} = i\sigma(n)(\nabla\phi_a)^2 - \phi_a\left(\dot{n} - \nabla\left(D(n)\nabla n\right)\right) + \cdots$$

[Martin Siggia Rose '73, …]

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Degrees of freedom: density and associated noise field

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'Wilsonian coefficients' of the EFT

Higher derivative terms, e.g.

$$j_i = -D\nabla_i n + a_{0,1}\nabla^2\nabla_i n + \cdots$$

and higher order in ϕ_a terms

EFT for fluctuating hydrodynamics

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$$G^R_{nn}(\omega, k) = \frac{\chi D k^2}{D k^2 - i\omega} + \cdots$$

EFT for fluctuating hydrodynamics

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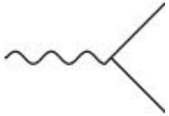
Leading nonlinearities come expanding $\sigma(n) = \sigma + \sigma'\delta n + \ldots\,,\; D(n) = D + D'\delta n + \ldots$

$\Rightarrow$ Nonlinear response tied to linear response!

$$\mathcal{L}^{(3)} \supset \frac{1}{2}D'\nabla^2\phi_a n^2$$



LD Mishra '23

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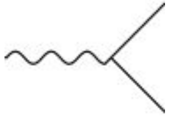
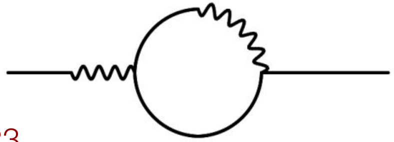


LD Mishra '23

… as are corrections to transport!

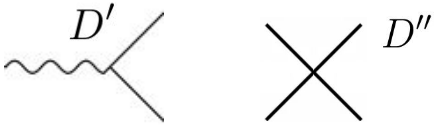


Michailidis Abanin LD '23

Precision tests of the EFT

Tying linear response to nonlinear response: [Mishra LVD ‘23]

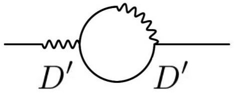
$D(n) \quad \Rightarrow$

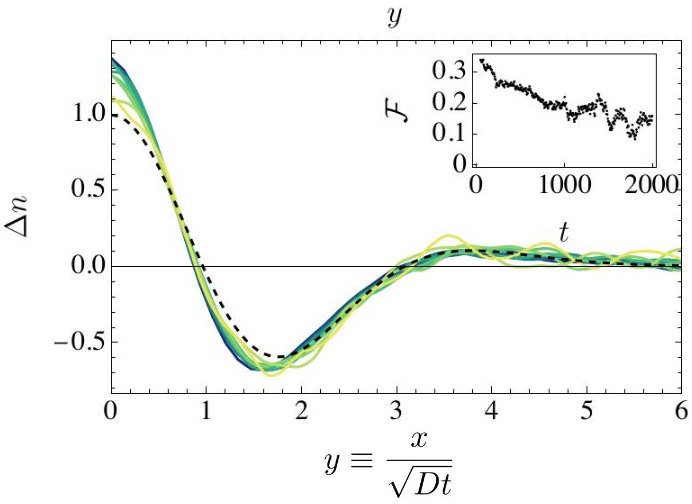


$\cdots$

Corrections to transport: [Michailidis Abanin LVD ‘23]

$$\langle n(x,t)n\rangle = \frac{1}{\sqrt{t}}\left[e^{-x^2/4Dt} + \frac{1}{\sqrt{t}}F_{1,0}\left(\frac{x^2}{Dt}\right) + \cdots\right]$$





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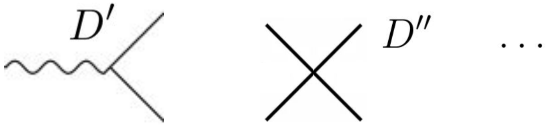
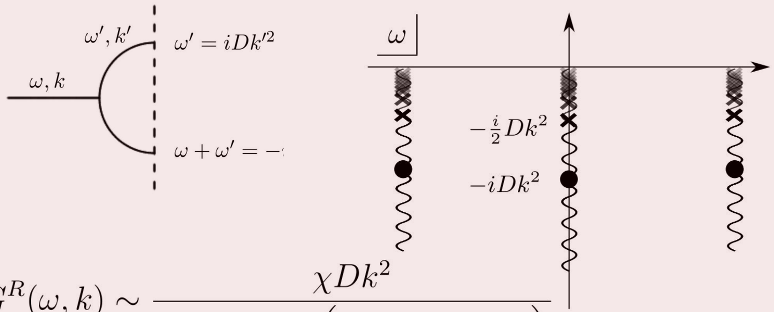


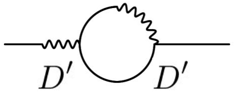
"Two-diffuson" branch cut structure

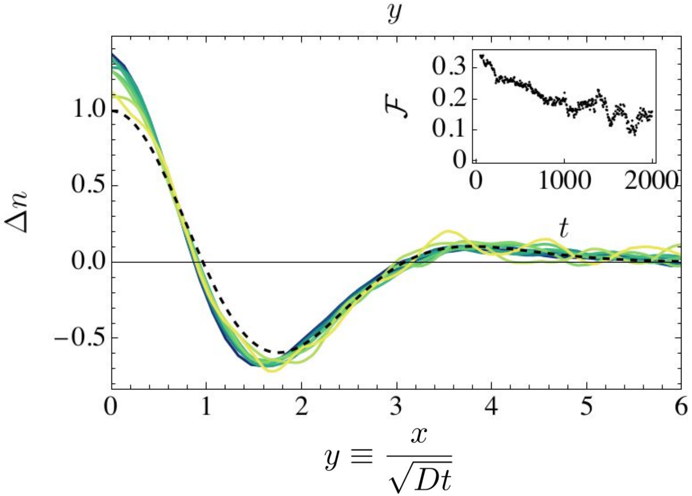


$$G^R(\omega, k) \sim \frac{\chi D k^2}{-i\omega + Dk^2\left(1 + \frac{D'^2 i\omega}{\sqrt{\frac{1}{2}Dk^2 - i\omega}}\right)}$$

Abanin LVD '23]

$$\left(\frac{x^2}{Dt}\right) + \cdots \Big]$$





Precision tests of the EFT

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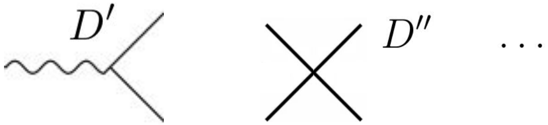

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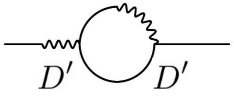
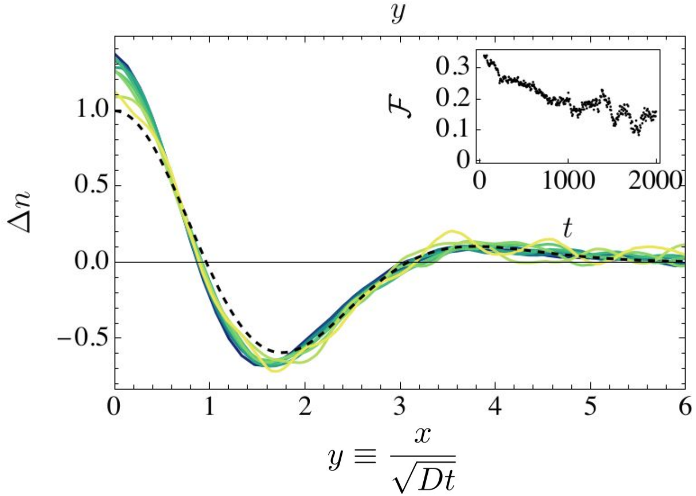




Strong coupling scale of hydrodynamics

[LVD '23]

Nonlinearity allows one to identify strong coupling scale of hydrodynamics

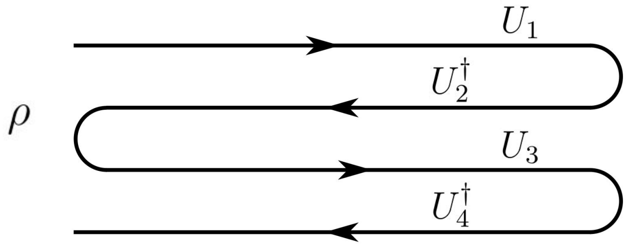
$$\tau_{\rm eq} \geq \frac{(T\chi)^{2/d}}{4\pi D}\left(\frac{D'}{D}\right)^{4/d}$$

In relativistic QFT, this leads to an almost-"Planckian" bound

$$\tau_{\rm eq} \gtrsim \frac{\hbar}{T}\frac{1}{(s_o)^{1/d}} \qquad (\, s = s_o T^{d+1}\,)$$

Out-of-time ordered transport

Symmetries of mixed state evolution



Generating functional:

$$Z_4[J_1, J_2, J_3, J_4] \equiv \mathrm{Tr}\left(\rho U_4^\dagger U_3 U_2^\dagger U_1\right) \qquad U_i = Te^{-i\int dt\, H(t) - J_i(t)\mathcal{O}(t)}$$

Object being time evolved is $\rho \otimes \mathbb{1} \; \in (\mathcal{H} \otimes \mathcal{H}^*)^2$

For quantum dynamics with symmetry group G , dynamics preserves quadrupled symmetry $G \times G \times G \times G$

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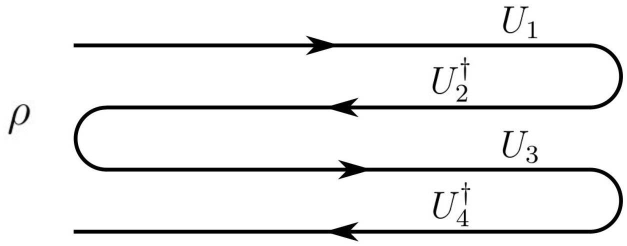

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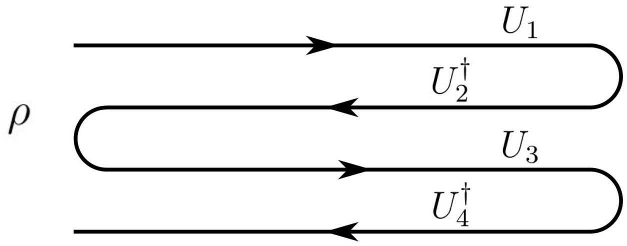
For quantum dynamics with symmetry group G, dynamics preserves quadrupled symmetry $G \times G \times G \times G$

Generalization of SWSSB:

$$\mathrm{Tr}\left(\rho\psi^\dagger\psi\psi^\dagger\psi\right) = \langle \psi_4^\dagger \psi_3 \psi_2^\dagger \psi_1 \rangle \neq 0 \quad \Rightarrow \quad G \times G \times G \times G \to G_{\mathrm{diag}}$$

Field content of the EFT



3 Goldstones and one Schwinger-Keldysh partner (=density)

Generalized “Keldysh basis”:

$$\phi_R = \phi_1 + \phi_2 + \phi_3 + \phi_4$$

$$\phi_A = \phi_1 - \phi_2 + \phi_3 - \phi_4$$

$$\phi_+ = \phi_1 + \phi_2 - \phi_3 - \phi_4$$

$$\phi_- = \phi_1 - \phi_2 - \phi_3 + \phi_4$$

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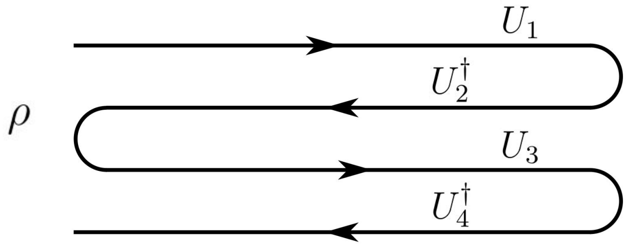

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~~$\phi_R = \phi_1 + \phi_2 + \phi_3 + \phi_4$~~ $\longrightarrow \quad \mu_R \sim \dot{\phi}_R$

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$$\phi_+ = \phi_1 + \phi_2 - \phi_3 - \phi_4$$

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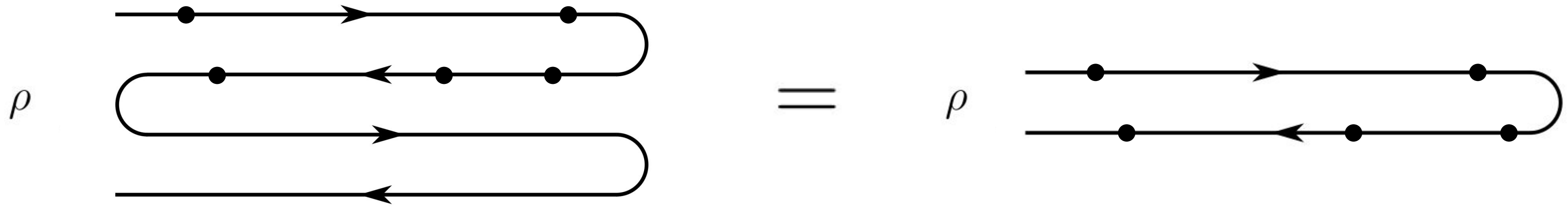
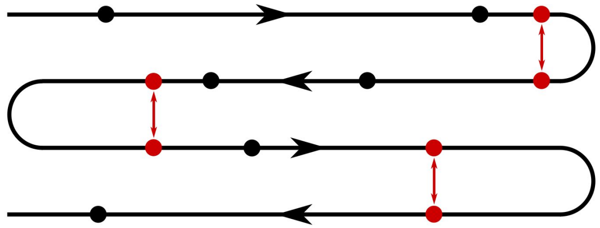
Many fields… but many constraints!

Constraints on the EFT

Any correlator involving only 2 types of fields can be obtained from original EFT



More generally, many “latest time” conditions:



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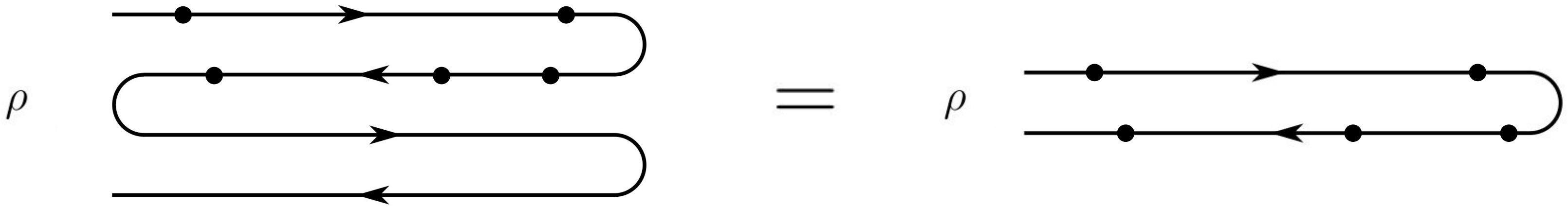

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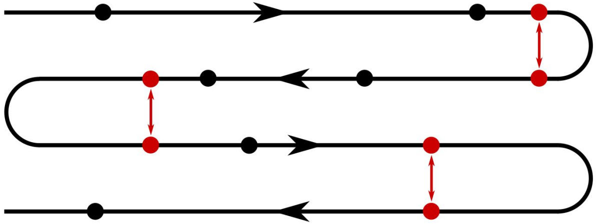
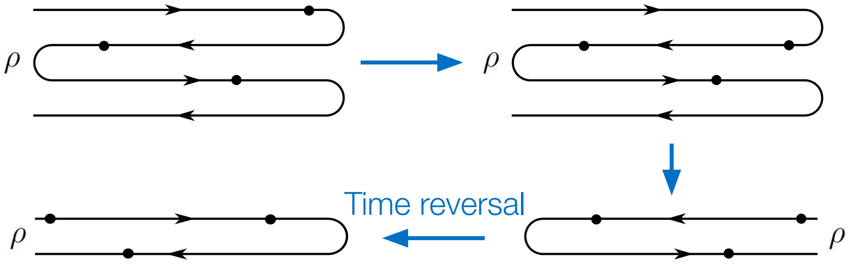


KMS also kills any <4pt function



Constraints on the EFT

$$Z_4[A_1, A_2, A_3, A_4] \equiv \mathrm{Tr}\left(\rho U_4^\dagger U_3 U_2^\dagger U_1\right)$$

Conditions:

1. $$Z_4[A_1, A_2, A, A] = Z_4[A_1, A, A, A_2] = Z_4[A, A, A_1, A_2] = Z_2[A_1, A_2]\,.$$ "collapse" rule

2. $$Z_4[A_1, A_2, A_3, A_4]^* = Z_4[A_4, A_3, A_2, A_1]$$ Unitarity

3. $$Z_4 = Z_4[A^1(-t+i\beta), A^4(-t), A^3(-t), A^2(-t)]\,,$$ $$Z_4 = Z_4[A^3(t), A^4(t), A^1(t+i\beta), A^2(t+i\beta)]\,.$$ KMS

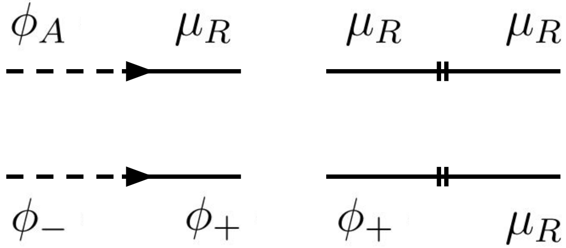
Look for actions satisfying these constraints

$$S_{(2)} = \frac{1}{2}\int_p M_{IJ}(p)\phi^I_{-p}\phi^J_p\,, \qquad S_{(3)} = \frac{1}{3!}\int_{p_1p_2p_3} \delta_{\Sigma_i p_i} M_{IJK}(\{p\})\phi^I_{p_1}\phi^J_{p_2}\phi^K_{p_3}\,, \qquad \ldots$$

EFT for OTOCs

The quadratic action for the "2-CTP" EFT to leading order in derivative is

$$S_{(2)} = \frac{\chi}{4}\int_{tx} 4iTD(\nabla\phi^A)^2 + \mu^R(\dot{\phi}^A + D\nabla^2\phi^A)$$
$$+ \dot{\phi}^+(\dot{\phi}^- + D\nabla^2\phi^-) + 2D\nabla\dot{\phi}^A\nabla\phi^- + \cdots$$



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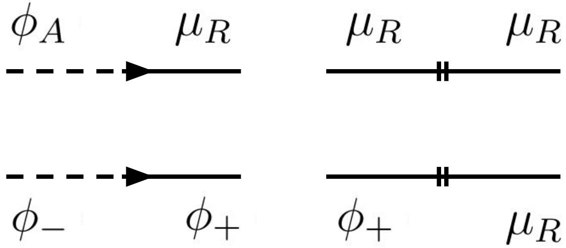

All cubic terms are also just 2-CTP incarnations of 1-CTP terms

$$S_{(3)} = \int \chi' O_{\chi'}(\phi) + D' O_{D'}(\phi) + \cdots$$

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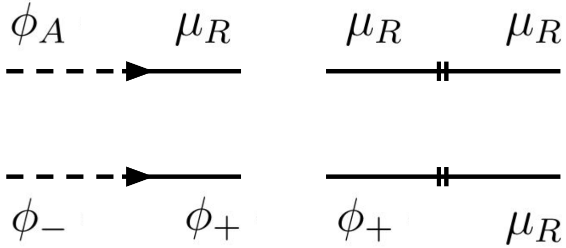

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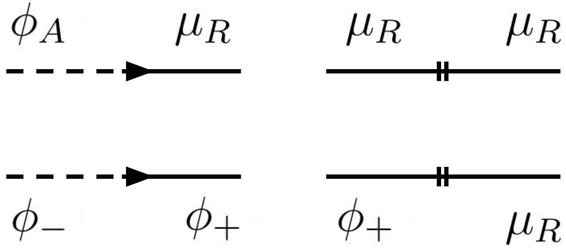
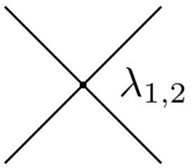


All cubic terms are also just 2-CTP incarnations of 1-CTP terms

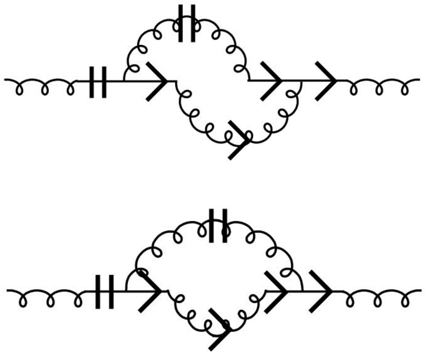
Two new quartic terms at leading order: $S_{(4)} = \int dt d^d x \, \lambda_1 \mathcal{L}_1 + \lambda_2 \mathcal{L}_2$

$$\mathcal{L}_1 = i\left[\dot{\phi}_-^2\left(\nabla\phi_+^2 - \nabla\phi_A^2\right) + \nabla\phi_-^2\left(\dot{\phi}_+^2 - \dot{\phi}_A^2\right) + \cdots\right]$$
$$\mathcal{L}_2 = i\left[\dot{\phi}_-\nabla\phi_-(\dot{\phi}_+\nabla\phi_+ - \dot{\phi}_A\nabla\phi_A) + \cdots\right]$$

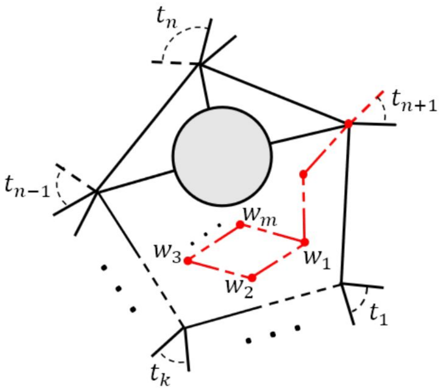


Causal structure of the EFT

"Flow of time" property simplifies diagramatics

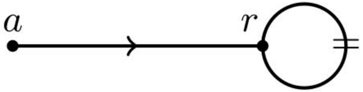


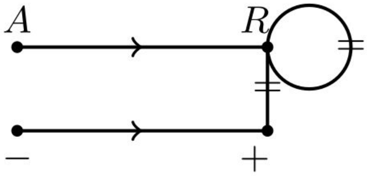
Caron-Huot Moore '08



Gao Glorioso Liu '18

Generalized structure in OTOC EFT:





Positivity bound on OTO-transport parameters?

The real time generating functionals satisfy

$$|Z| \leq 1$$

Glorioso Liu '16
Jensen Marjieh Pinzani-Fokeeva Yarom '18

It is often assumed that this requires $\operatorname{Im} S \geq 0$. However, that condition is too strong, and can lead to wrong constraints! (e.g., $\mathcal{L} \sim iT[\sigma(\nabla\phi_a)^2 + \alpha(\dot\phi_a^2)]$)

Nevertheless, a weaker constraint should exist, and be interesting for OTOCs:

$$\begin{aligned}\operatorname{Im} \mathcal{L}_{(4)}[\phi_A = 0] = \lambda_1 &\left[(\dot\phi_+ \nabla\phi_-)^2 + (\dot\phi_- \nabla\phi_+)^2\right] \\ &+ \lambda_2\, \dot\phi_+ \dot\phi_- (\nabla\phi_- \cdot \nabla\phi_-)\,.\end{aligned}$$

Positivity of this action leads to $\lambda_1 \geq \frac{1}{2}|\lambda_2| > 0$

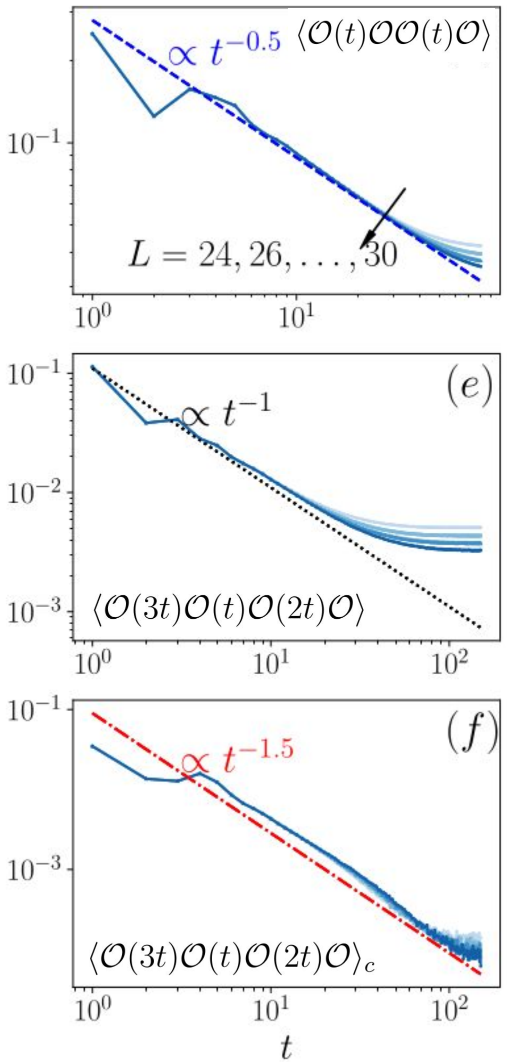
Comparison with numerics

Power-law behavior

One output of the EFT is that operators almost commute at late times $[A, B] \sim \frac{\hbar\omega}{\Lambda} AB$ (note $\Lambda \neq T$ in general)

Furthermore, densities are weakly coupled in hydrodynamics:

$$\langle \mathcal{O}(3t)\mathcal{O}(t)\mathcal{O}(2t)\mathcal{O}\rangle \simeq \langle \mathcal{O}(3t)\mathcal{O}(2t)\mathcal{O}(t)\mathcal{O}\rangle$$
$$\simeq \langle \mathcal{O}(t)\mathcal{O}\rangle^2 \sim 1/t$$



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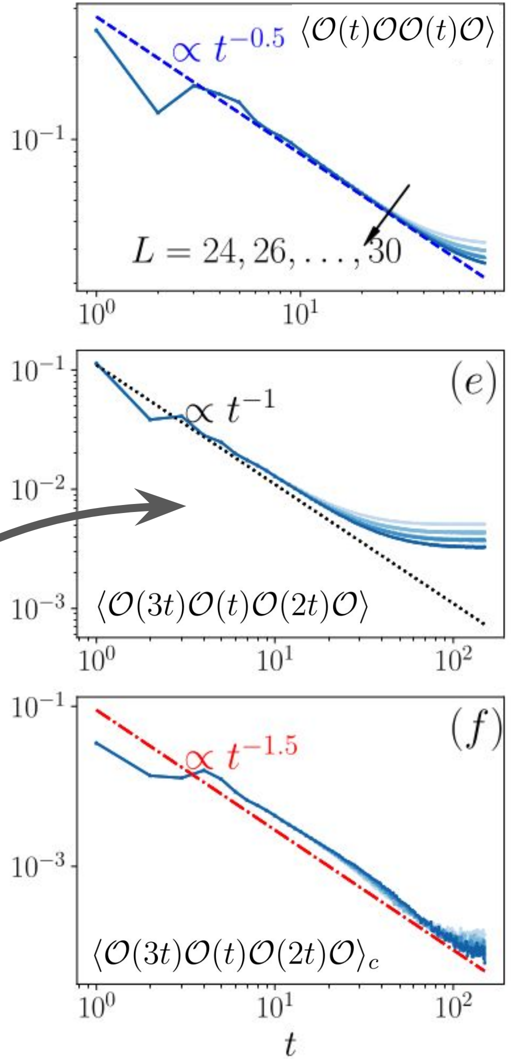

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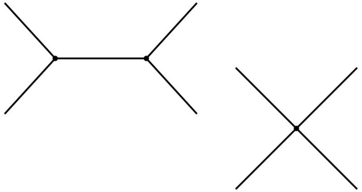
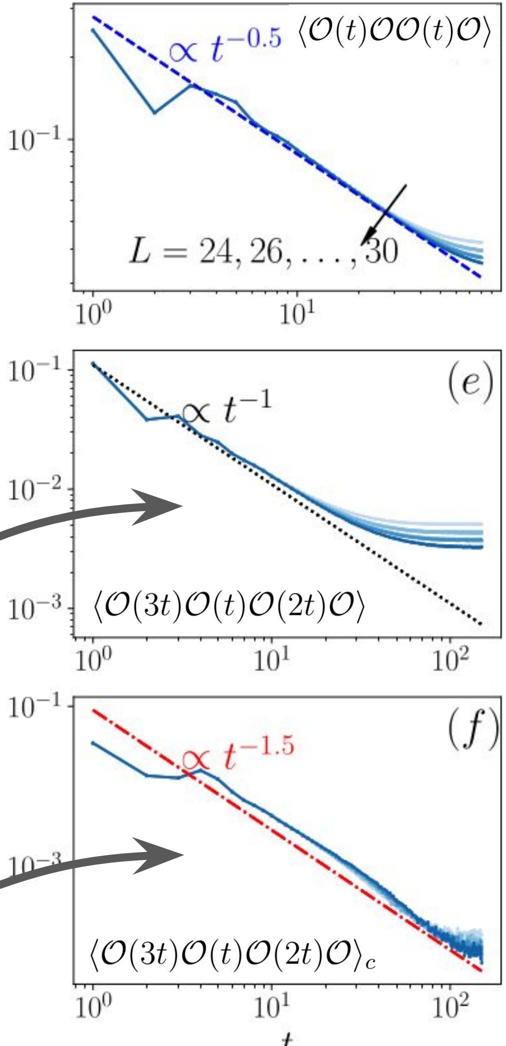
This invites to look at connected correlators. Requires two cubic vertices each suppressed by $\delta n \sim 1/t^{1/4}$



$$\langle \mathcal{O}(3t)\mathcal{O}(t)\mathcal{O}(2t)\mathcal{O}\rangle_c \sim 1/t^{3/2}$$



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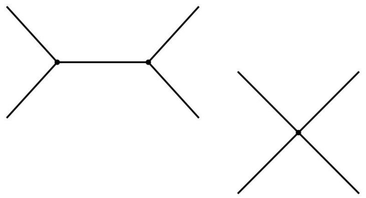

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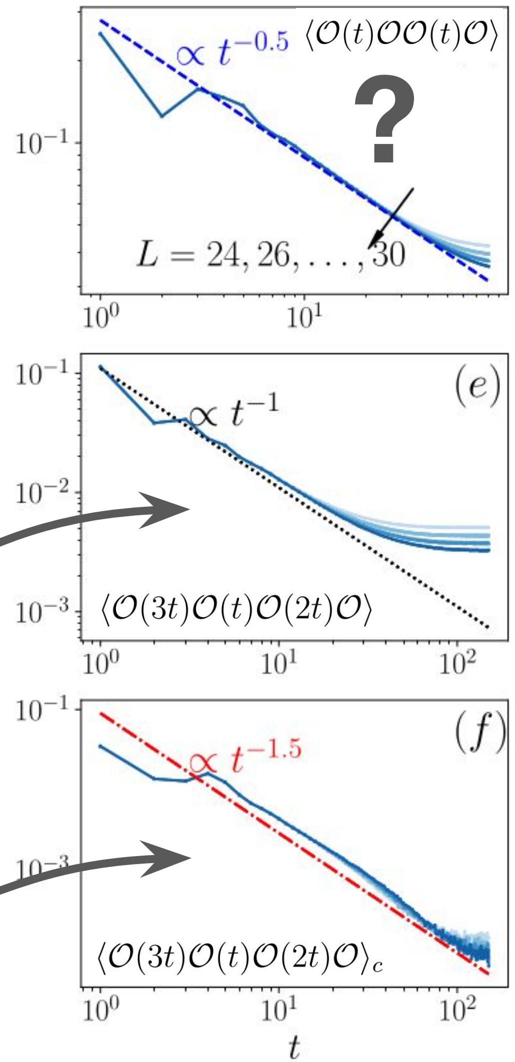


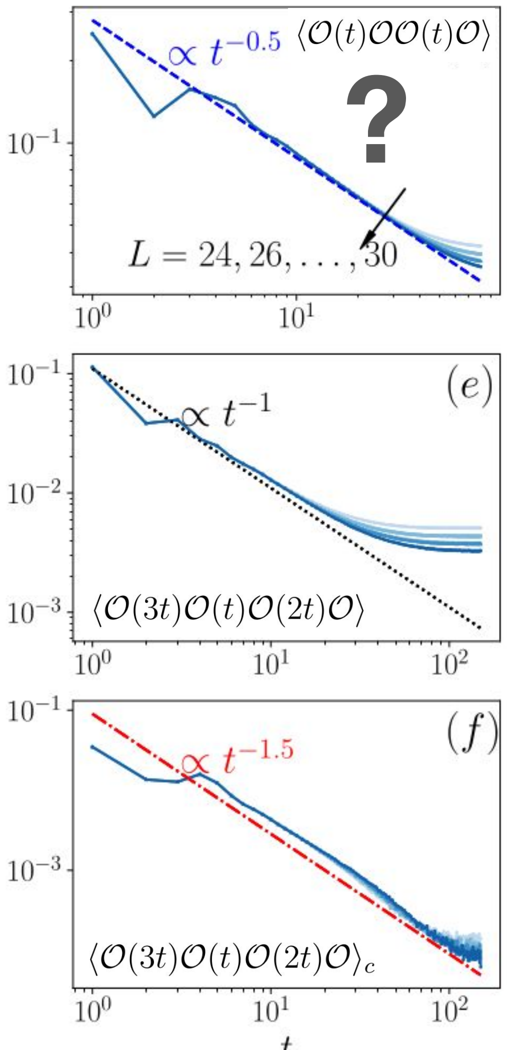
Power-law behavior

Coincident operators are singular from the EFT perspective, must resolve using operator product expansion:

$$\mathcal{O}\mathcal{O} \sim \alpha_1 \delta n + \alpha_2 \nabla^2 \delta n + \alpha_3 (\delta n)^2 + \cdots$$

At leading order:

$$\langle \mathcal{O}(t)\mathcal{O}\mathcal{O}(t)\mathcal{O} \rangle \simeq (\alpha_1)^2 \langle \delta n(t) \delta n \rangle \sim 1/\sqrt{t}$$



OTO-transport

New physics hides in **difference** between OTOCs and TOCs

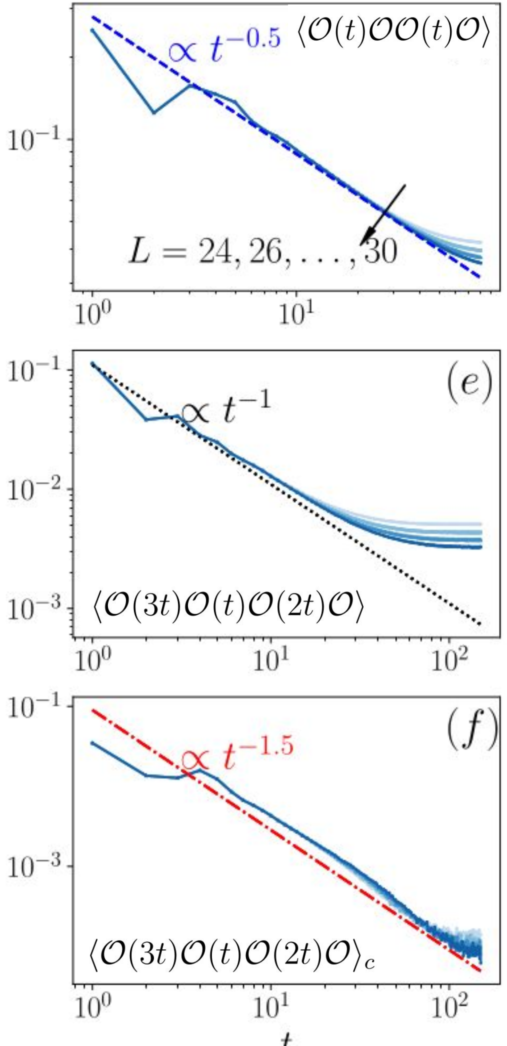
Trace cyclicity ⇒ 3 independent 4pt functions

[Wang Heinz '98]
[Haehl Loganayagam Narayan Nizami Rangamani '17]

$$g_0 = \mathrm{Tr}\left(\rho O(t_4) O(t_3) O(t_2) O(t_1)\right)$$
$$g_1 = \mathrm{Tr}\left(\rho O(t_3) O(t_4) O(t_2) O(t_1)\right)$$
$$g_2 = \mathrm{Tr}\left(\rho O(t_4) O(t_2) O(t_3) O(t_1)\right)$$

Scaling in the EFT $\mu^R \sim \phi^A \sim \dot{\phi}^- \sim \phi^+ \sim q^{d/2}$ implies:

$$g_0 \sim \frac{1}{t^{3/2}} + \frac{i}{t^{5/2}}$$
$$g_1 - g_0 \sim \frac{1}{t^{7/2}} + \frac{i}{t^{5/2}}$$
$$g_2 - g_0 \sim \frac{1}{t^{7/2}} + \frac{i}{t^{5/2}}$$



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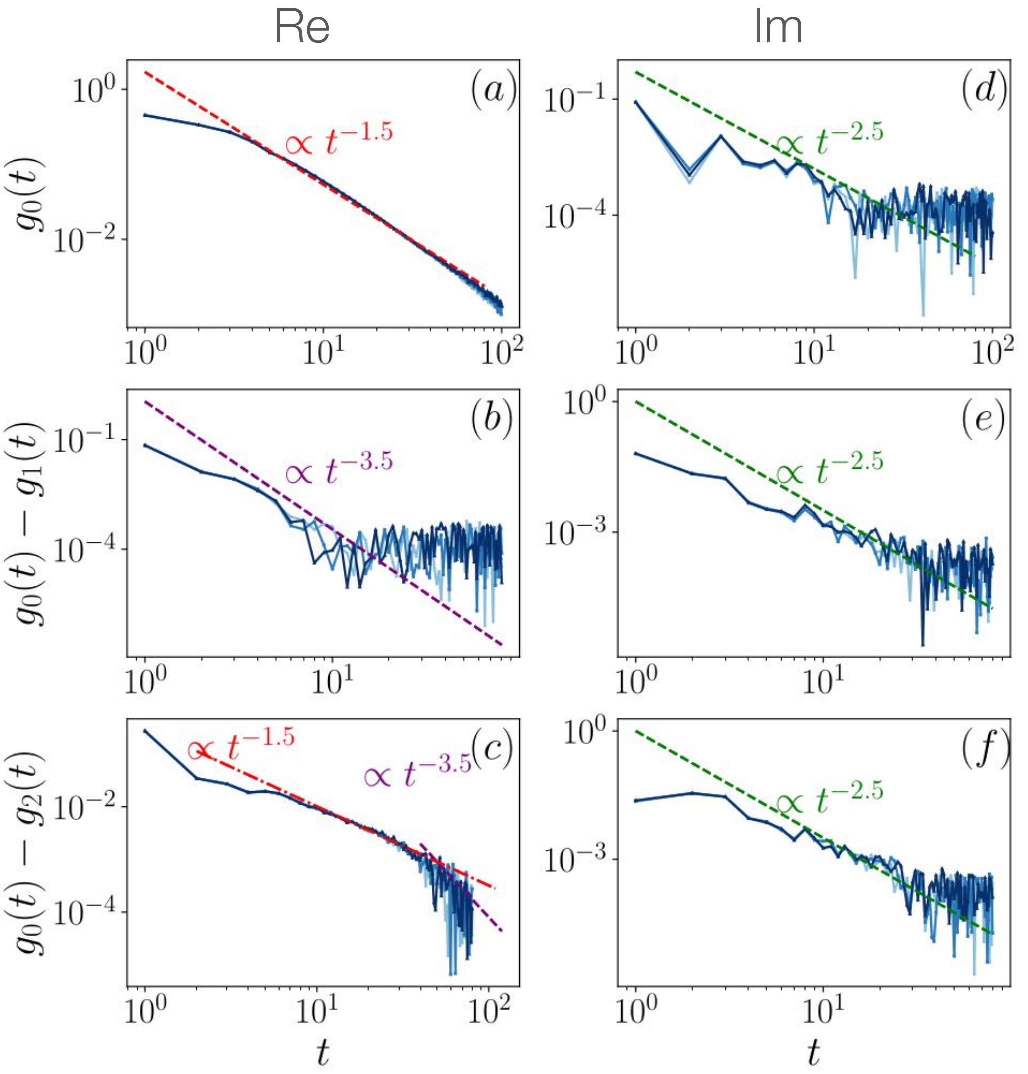

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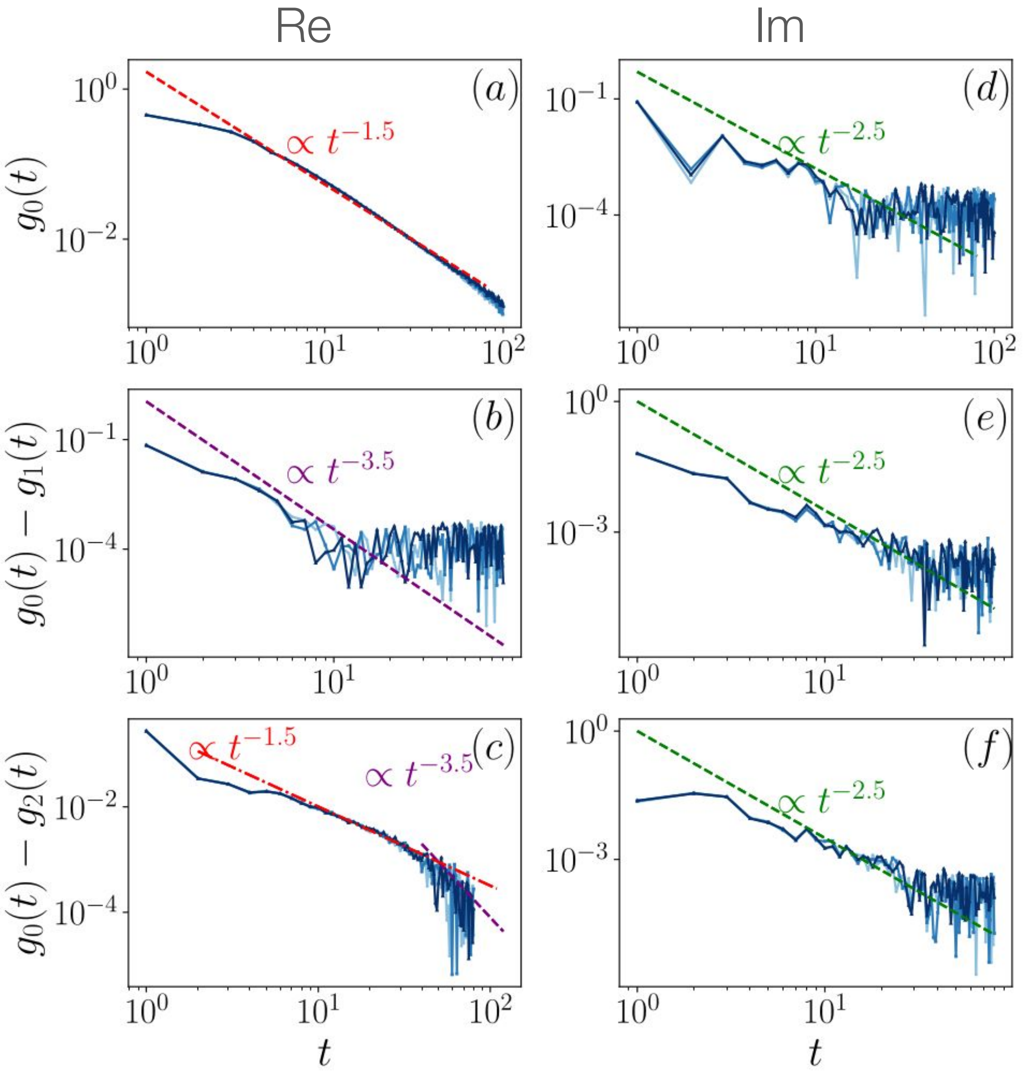
$\lambda_1,\ \lambda_2$



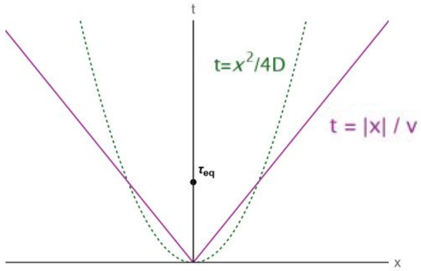
Connection to previous work

Extensive literature on OTOCs – early work focused on the "butterfly effect" and Lyapunov growth [Larkin Ovchinnikov '69, Maldacena Roberts Shenker Stanford Susskind '14-'15, Aleiner Faoro Ioffe '16, …]

$$\langle \mathcal{O}(t,x)\mathcal{O}\mathcal{O}(t,x)\mathcal{O}\rangle \sim 1 - \frac{1}{N}e^{\lambda_L(t-\frac{|x|}{v_B})} + \cdots$$

which only arises in semiclassical / large N models. Butterfly cones seem to be more generic [Khemani Huse Nahum '18, Rakovszky Pollmann Keyserlingk '18, Khemani Vishwanath Huse '18, Keselman Nie Berg '20, …]

EFTs for this front have been put forward ("information dynamics") [Xu Swingle '19, Gao Liu '23, Choi Haehl Mezei Sarosi '23]



Would be nice to understand interplay with our phenomena. However, one EFT seems to break down in another's regime.

While butterfly cones are sensitive to noise [Jacoby Huse Gopalakrishnan '24], I suspect that our signatures are not.

Back up slides

Resulting EFT

With particle-hole symmetry $n \to -n$, the 2-legged EFT is schematically

$$S[\phi_a, \mu_r] = \int \chi O(\phi^2) + \sigma O(\phi^2) + \chi'' O(\phi^4) + \sigma'' O(\phi^4) + \cdots$$

Whereas the 4-legged EFT is

$$\begin{aligned} S[\phi_A, \phi_+, \phi_-, \mu_R] = &\int \chi O(\phi^2) + \sigma O(\phi^2) + \chi'' O(\phi^4) + \sigma'' O(\phi^4) \\ &+ \int \lambda_1 O(\phi^4) + \lambda_2 O(\phi^4) + \cdots \end{aligned}$$

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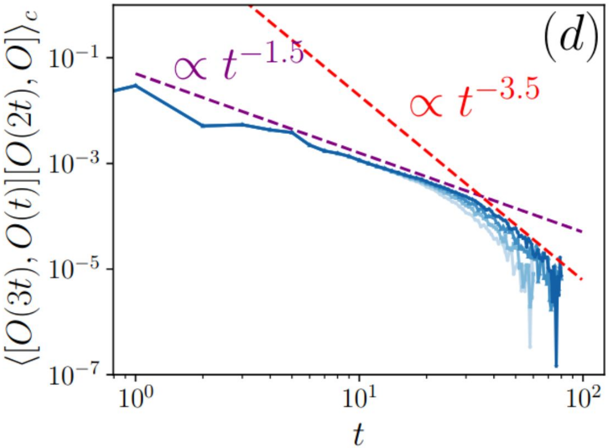

2-CTP EFT

Gaussian action:

$$S = \frac{\chi}{4}\int \dot{\phi}_R\dot{\phi}_A + \dot{\phi}_+\dot{\phi}_- + D\nabla\phi_A\left(iT\nabla\phi_A + \nabla\dot{\phi}_R + 2\nabla\dot{\phi}_-\right) + D\nabla\phi_-\nabla\dot{\phi}_+$$

Example quartic term: $$S_{\chi''}[\phi_{1234}] = \frac{\chi''}{4^2 4!}\int \dot{\phi}_A\dot{\phi}_R^3 + \dot{\phi}_+\dot{\phi}_-^3 + 3\dot{\phi}_A\dot{\phi}_R\dot{\phi}_-^2 + 3\dot{\phi}_+\dot{\phi}_-\dot{\phi}_R^2 + \dot{\phi}_+^3\dot{\phi}_- + 3\dot{\phi}_+^2\dot{\phi}_A\dot{\phi}_R + \left(\dot{\phi}_A^3\dot{\phi}_R + 3\dot{\phi}_A^2\dot{\phi}_+\dot{\phi}_-\right)$$

Compare to 1-CTP version: $$S_{\chi''}[\phi_a, \phi_r] = \frac{1}{4!}\chi''\int \dot{\phi}_a\dot{\phi}_r^3 + 4\dot{\phi}_a^3\dot{\phi}_r$$