COSET INSIGHTS INTO HYDRODYNAMIC EFFECTIVE THEORY

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SYMMETRIES AND HYDRODYNAMICS

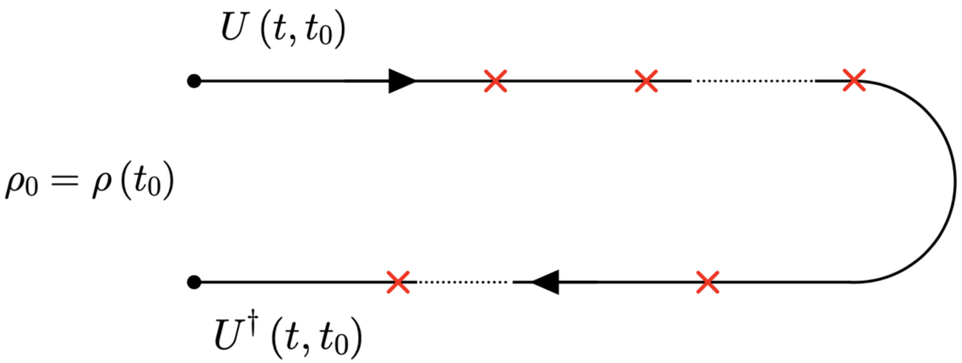
- System described by finite temperature density matrix → long-distance behavior is hydrodynamics
- Most fluctuations have correlation length order of Mean Free Path (MFP)
- Only those quantities conserved due to symmetry can be transported
  - Energy, momentum, internal symmetry charges
- For simplicity and time will only discuss transport of internal symmetries – **however**, parallel arguments can be made for "real hydrodynamics"

SCHWINGER-KELDYSH PATH INTEGRAL

- Since non-relativistic, long-distance excitations can dissipate into ones shorter than MFP (heat)
- EFT must be non-unitary → need an unusual formalism
- Schwinger-Keldysh contour, to calculate time-unordered correlations, is the right tool for the job

$$\mathrm{Tr}(\,\bar{T}(\cdots)\,T(\cdots)\,\rho_0)$$

$$\mathcal{Z}[J_1, J_2] = \int_{BC} \mathcal{D}\Phi_1 \mathcal{D}\Phi_2 \, e^{iS(\Phi_1) - iS(\Phi_2) + i\int J_1 \cdot \mathcal{O}_1 - i\int J_2 \cdot\cdot \mathcal{O}_2}$$



SCHWINGER-KELDYSH PATH INTEGRAL

- Convenient to use variables that live on a single time contour
- Should think of r-fields as "classical" and a-fields as "quantum" fluctuations
  - There is a power counting argument to back this up

$$\Phi_r = \tfrac{1}{2}(\Phi_1 + \Phi_2)$$

$$\Phi_a = \Phi_1 - \Phi_2$$

$$\begin{aligned}
\langle \mathcal{O}_r(x)\mathcal{O}_r(y)\rangle_{\rho_0} &= \frac{1}{2}\langle\{\mathcal{O}(x),\mathcal{O}(y)\}\rangle_{\rho_0} \ , \\
\langle \mathcal{O}_r(x)\mathcal{O}_a(y)\rangle_{\rho_0} &= i\theta(x^0-y^0)\langle[\mathcal{O}(x),\mathcal{O}(y)]\rangle_{\rho_0} \ , \\
\langle \mathcal{O}_a(x)\mathcal{O}_r(y)\rangle_{\rho_0} &= -i\theta(y^0-x^0)\langle[\mathcal{O}(x),\mathcal{O}(y)]\rangle_{\rho_0} \ , \\
\langle \mathcal{O}_a(x)\mathcal{O}_a(y)\rangle_{\rho_0} &= 0 \ .
\end{aligned}$$

KUBO-MARTIN-SCHWINGER RELATIONS (KMS)

- The special form of the density matrix (built from the Hamiltonian) leads to a symmetry of the generating functional
- Gives rise to the fluctuation-dissipation theorem (will see example later)
- Looks like a boost in the r/a-variables
  - Usually truncated to classical limit (recall a-fields go as $\hbar\beta$) – but our results hold at all orders

Note unusual expansion in ω/T

$$J_1^{\Theta}(x^\mu) = \eta_\Theta\, J_1\left(-x^\mu + i\frac{\beta^\mu}{2}\right) \,, \qquad J_2^{\Theta}(x^\mu) = \eta_\Theta\, J_2\left(-x^\mu - i\frac{\beta^\mu}{2}\right)$$

$$\begin{pmatrix} J_r^{\Theta}(x^\mu) \\ J_a^{\Theta}(x^\mu) \end{pmatrix} = \eta_\Theta \begin{pmatrix} \cosh\left(i\frac{\beta}{2}\partial_t\right) & \frac{1}{2}\sinh\left(i\frac{\beta}{2}\partial_t\right) \\ 2\sinh\left(i\frac{\beta}{2}\partial_t\right) & \cosh\left(i\frac{\beta}{2}\partial_t\right) \end{pmatrix} \begin{pmatrix} J_r(y^\mu) \\ J_a(y^\mu) \end{pmatrix} \Bigg|_{y=-x}$$

Classical limit like Galilean boosts

SCHWINGER-KELDYSH EFFECTIVE THEORY

- Now want the theory of long-distance fluctuations
- Let the sources be background gauge fields (Abelian for simplicity)
- "Integrating in" Stückelberg fields gives gauge invariance
- Also need UV unitarity, and dynamical KMS* (DKMS, the extension of KMS to the fields)

*Unusual: Changes contour. Can show is OK

$$\mathcal{Z}[A_{r\mu}, A_{a\mu}] = \int \mathcal{D}\phi_r \mathcal{D}\phi_a \, e^{iS_{\mathrm{EFT}}[A_{r\mu}+\partial_\mu\phi_r,\, A_{a\mu}+\partial_\mu\phi_a]}$$

UNITARITY CONSTRAINTS

- UV unitarity constrains form even more: Proportional to a-fields, a-even terms imaginary and their sum positive
- In summary: Gauge invariance, Unitarity, KMS

$$S_{\mathrm{EFT}}[\phi_r, \phi_a = 0, A_{\mu r}, A_{\mu a} = 0] = 0$$

$$S^*_{\mathrm{EFT}}[\phi_r, \phi_a, A_{\mu r}, A_{\mu a}] = -S_{\mathrm{EFT}}[\phi_r, -\phi_a, A_{\mu r}, -A_{\mu a}]$$

$$\mathrm{Im}\, S_{\mathrm{EFT}}[\phi_r, \phi_a, A_{\mu r}, A_{\mu a}] \geq 0$$

TIME-INDEPENDENT GAUGE SYMMETRY

- Consider the quadratic action for U(1) charge transport (background gauge fields not written) in the classical limit, with DKMS giving fluctuation-dissipation
- Dispersion relation has both propagating and diffusive parts
- In order to obtain only diffusive (correct) behavior, must impose gauge symmetry for r-field

Glorioso, Liu, 1805.09331
and refs. therein

$$0 = \omega^2 - c_s^2 k^2 + i\frac{\sigma}{\chi}\omega k^2$$

$$S_{\mathrm{EFT}} = \int dt\, d^3\mathbf{x} \left[\chi\left(\dot{\phi}_a\dot{\phi}_r - c_s^2\nabla\phi_a\cdot\nabla\phi_r\right) + \sigma\left(-\nabla\phi_a\cdot\nabla\dot{\phi}_r + \frac{i}{\beta}(\nabla\phi_a)^2\right)\right]$$

$$\phi(t,\mathbf{x}) \to \phi(t,\mathbf{x}) + f(\mathbf{x}) \implies c_s^2 = 0$$

Non-unitary: dissipative!

Diffeomorphisms

ASIDE: POWER COUNTING

$$\frac{S}{\hbar} = \frac{p_*}{\hbar}\int dt\, d^3\mathbf{x}\left[\frac{k_*^2}{\omega_*}\dot{\phi}_a\dot{\phi}_r - \nabla\phi_a\nabla\dot{\phi}_r + \frac{i}{\beta\hbar}(\nabla\phi_a)^2\right]$$

$$\frac{\mathrm{Im}\ S}{\hbar} \lesssim 1$$

New

$$\frac{\mathrm{Re}\ S}{\hbar} \sim 1$$

$$\frac{\phi_a}{\phi_r} \lesssim \frac{\hbar\omega}{T}$$

In addition to MFP/T derivative expansion!

TIME-INDEPENDENT GAUGE SYMMETRY

- What is the origin of this time-independent gauge symmetry?
  - Previously justified by defining a fluid to have invariance under phase shifts at each point in space
- Is it a necessary ingredient of the effective theory, particularly in the case of energy and momentum transport (the famous time-independent diffeomorphisms)?
- To answer these questions, we will employ the **coset construction**

Akyuz, Goon, Penco, 2306.17232

$$\rho_0 \to g_1 \rho_0 g_2^{-1}$$

$$G_1 \times G_2 \to G_r$$

COSET CONSTRUCTION

- Symmetry is **doubled** in along contour, only density matrix breaks to diagonal → spontaneous breaking (sometimes called “strong to weak” breaking)
- EFT ingredients are Maurer-Cartan form of the coset, and any matter fields in representation of unbroken group (we are forced to take one in adjoint to partner with Goldstone)

$$\Omega = e^{i\phi_a \cdot T_a} \qquad \omega = -i\Omega^{-1}\partial_\mu \Omega \equiv D_\mu \phi_a \cdot T_a + \mathcal{A}_\mu \cdot T_r$$

$$D_\mu \phi_a, \qquad \nabla_\mu = \partial_\mu + i\left[\mathcal{A}_\mu\,,\,\cdot\,\right], \qquad \rho_r$$

β, u^μ

REDUNDANT CONSTRUCTION

- But how to implement DKMS symmetry relating Goldstone to matter field?
- Idea: add another field to the coset (denoted by tilde)
  - But need a redundancy to maintain the right properties (3 options)
- To commute with KMS, redundancy must be time-independent!

Akyuz, Goon, Penco, 2306.17232

$$\tilde{\Omega} = e^{i\phi_a \cdot T_a} e^{i\phi_r \cdot T_r} \qquad \tilde{\Omega} \to \tilde{\Omega} h(x) \ , \qquad h(x) = e^{if(x)\cdot T_r}$$

$$\begin{Bmatrix} e^{i\phi_1(x)\cdot T} \\ e^{i\phi_2(x)\cdot T} \end{Bmatrix} \xrightarrow{\text{redundancy}} \begin{Bmatrix} e^{i\phi_1(x)\cdot T} e^{if(x)\cdot T} \\ e^{i\phi_2(x)\cdot T} e^{if(x)\cdot T} \end{Bmatrix} \xrightarrow{\text{DKMS}} \begin{Bmatrix} e^{i\phi_1\left(-x^\mu + i\frac{\beta^\mu}{2}\right)\cdot T} e^{if\left(-x^\mu + i\frac{\beta^\mu}{2}\right)\cdot T} \\ e^{i\phi_2\left(-x^\mu - i\frac{\beta^\mu}{2}\right)\cdot T} e^{if\left(-x^\mu - i\frac{\beta^\mu}{2}\right)\cdot T} \end{Bmatrix}$$

New

$$\beta^\mu \partial_\mu f = 0$$

$$\tilde{\Omega} = e^{i\phi_a \cdot T_a} e^{i\phi_r \cdot T_r}$$

MATCHING THE CONSTRUCTIONS

- DKMS can be defined to all orders in the redundant construction, now match to "standard" coset construction
- All "tilded" building blocks can be written in terms of "untilded" ones, up to exponentials – but since all ingredients adjoint → cancel with invariant action!

$$\tilde{\omega} \equiv \tilde{D}_\mu \phi_a \cdot T_a + \tilde{D}_\mu \phi_r \cdot T_r$$
$$= e^{-i\phi_r \cdot T_r} \Big[D_\mu \phi_a \cdot T_a + D_\mu \phi_r \cdot T_r \Big] e^{i\phi_r \cdot T_r}$$

$$D_\mu \phi_r \cdot T_r \equiv i e^{i\phi_r \cdot T_r} \partial_\mu e^{-i\phi_r \cdot T_r} + \mathcal{A}_\mu \cdot T_r$$

$$\rho_r \equiv D_0 \phi_r$$

$$\tilde{X} \cdot T = e^{-i\phi_r \cdot T_r} X \cdot T e^{i\phi_r \cdot T_r}$$

ALL-ORDERS DMKS

- Matching allows to define **new** all-orders DKMS transformations for non-redundant variables (*in the sense that they guarantee KMS of effective action)

$$\rho_r'(x) = -\cosh\left(\frac{i\beta}{2}\mathcal{D}_t\right)\rho_r(y) - \frac{1}{2}\sinh\left(\frac{i\beta}{2}\mathcal{D}_t\right)D_0\phi_a(y)\Bigg|_{y=-x} \qquad \mathcal{D}_t = \nabla_t - i\,[\,\rho_r\,,\,\cdot\,]$$

$$D_\mu\phi_a'(x) = -\cosh\left(\frac{i\beta}{2}\mathcal{D}_t\right)D_\mu\phi_a(y) - i\beta F\left(\frac{i\beta}{2}\mathcal{D}_t\right)\left(\mathcal{F}_{0\mu}(y) + \nabla_\mu\rho_r(y)\right)\Bigg|_{y=-x} \qquad F(z) = \sinh(z)/z$$

ALL-ORDERS EQUIVALENCE

- To summarize, have a dictionary relating the redundant and non-redundant (typical) coset building blocks
- Furthermore, works to all orders in DKMS
- Since all building blocks are adjoint, equivalence extends to action
- Finally, can demonstrate equivalence of ϕ_r Haar measure and ρ measure in path integral
- Conclusion: theory can be written in a gauge invariant way

REDUNDANT UTILITY

- Have shown that the effective theory can be framed entirely without redundant variables
- Might there nevertheless be some use for it? Yes.
- Separability of non-dissipative action + redundancy → manifest infinite dim. true symmetry

$$\mathcal{L}_1 = \chi \tilde{D}_0\phi_r \cdot \tilde{D}_0\phi_a = \frac{\chi}{2}\left[(\tilde{D}_0\phi_1)^2 - (\tilde{D}_0\phi_2)^2\right]$$

$$J^\mu = \chi\, \tilde{D}_0\phi_r\, \delta^\mu_0$$

Kelvin's theorem

CONCLUSIONS

- Schwinger-Keldysh approach provides an action formalism for effective theory of hydrodynamics with fluctuation and dissipation
  - Hydrodynamics is in fact a result of Goldstone theorem!
- Has required a “time-independent gauge symmetry” with formerly obscure origin
  - Coset construction makes origin quite apparent
  - Furthermore, while useful, coset shows that it is not necessary:
  - Non-redundant path integral is equivalent
- Future applications: hydrodynamic phenomena (e.g. finite temperature superfluid), condensed matter systems with emergent symmetries