Chimeric states of matter:

Meissner effect without superconductivity

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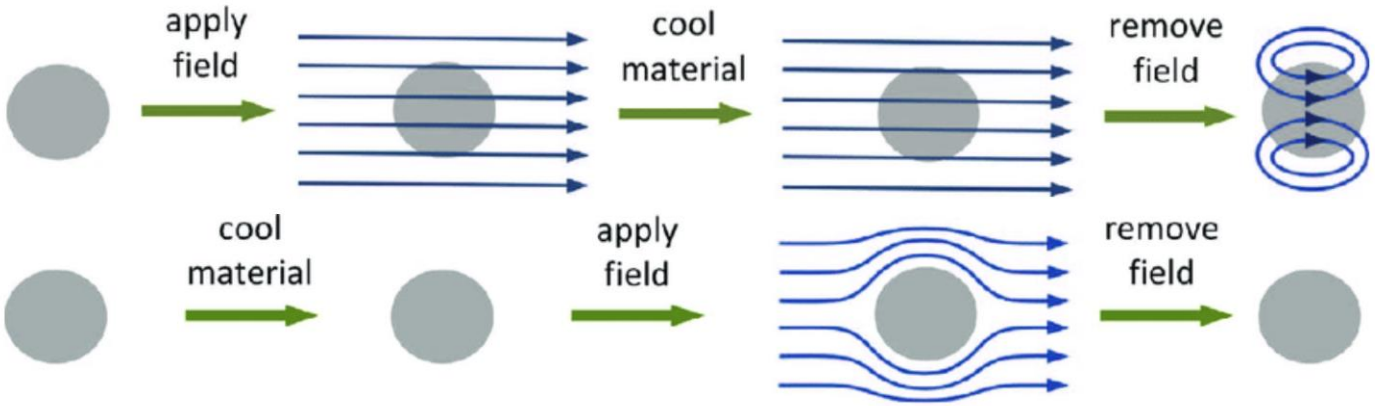
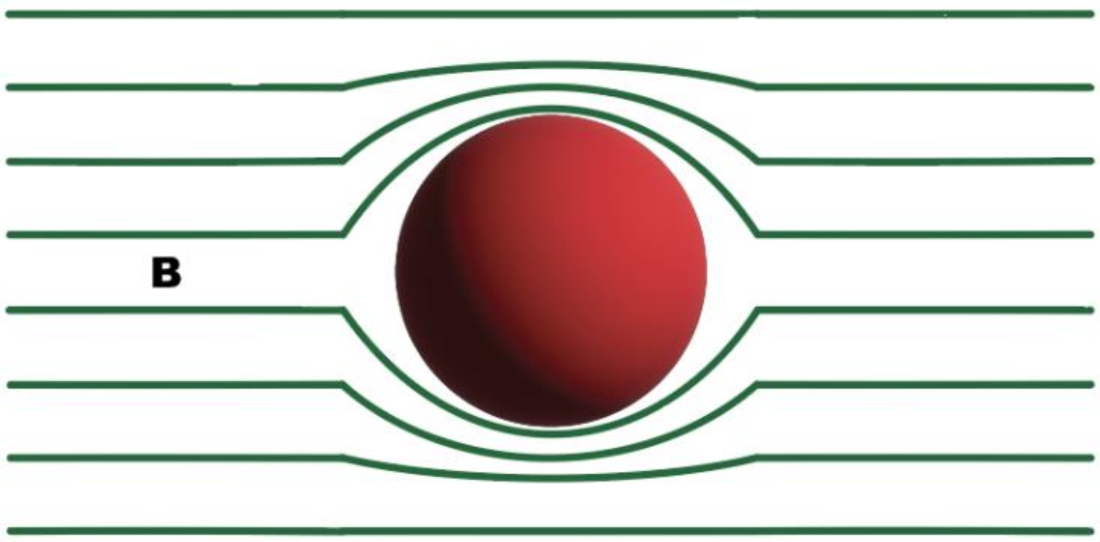
Perfect conductor

- **Infinite conductivity**
- Changes in B-field induce current flow
- Current never dies
- Result: **Locked in B-field**
- If B=0 at formation, remains for all time
- But non-zero B remains fixed too



Superconductor

- **Perfect conductor** + **Meissner**
- Meissner effect = **total expulsion of B**
- If B non-zero at formation, still expelled fully



Expectations

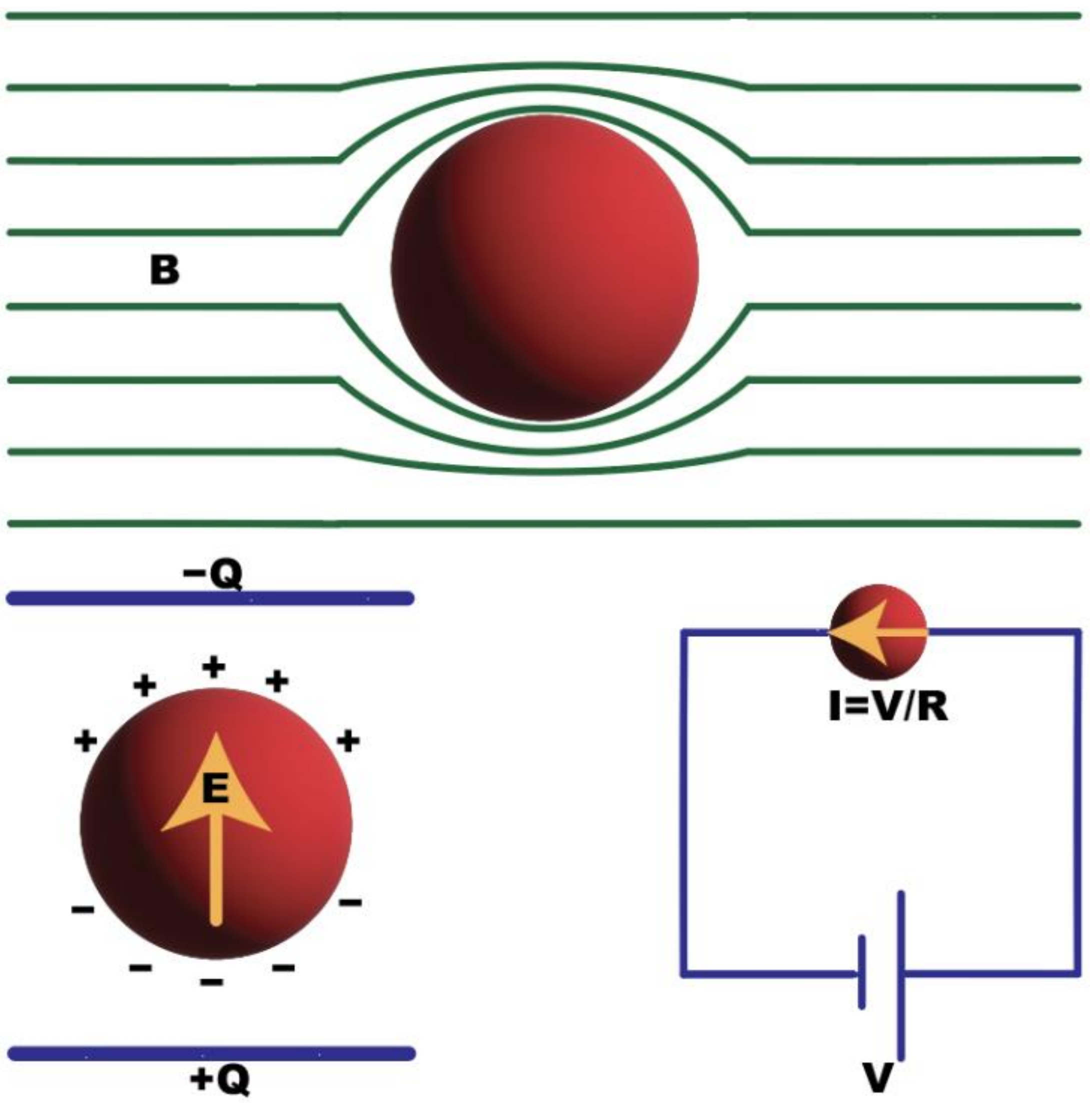
- **Superconductor** $\subset$ **perfect conductor** (True)
- Meissner effect requires **dissipationless current** flow (True)
- **Meissner effect** $\subset$ **perfect conductor** (False)

Chimeras

- Greek Mythology
- Unholy mixture of animals
- Lion and goat
- Preditor and prey

Chimeric conductors and insulators

- Novel theoretical state of matter
- Full Meissner effect (B totally expelled)
- Ohm's law: V=IR
- Unholy mixture of ordinary and superconducting responses
- How is this possible?



Theoretical difficulties

- Electromagnetic U(1) **symmetry either broken or unbroken**
- Superconductivity: **Broken** (Anderson Higgs Mechanism)
- Ordinary conductor: **Unbroken**
- Apparently **no middle-ground** that can interpolate
- Need **dissipationless current** flow and **Ohm's law** (contradiction?)

Theoretical solution

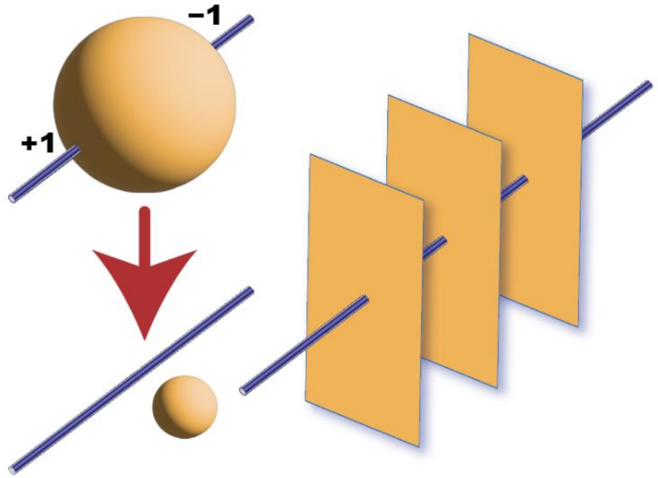
- **Generalize** notion of **SSB**
- Find **middle-ground** between broken and unbroken
- Requirement: **Generalized symmetries and mixed anomalies**
- **Magnetic response:** dissipatonless current
- **Electric response:** Ohm's law

Spontaneous symmetry breaking

- Broken U(1) ⇒ Goldstone boson

$$\phi \in [0, 2\pi)$$

- Jumps from 0 to 2π or vice versa create **winding planes**
- **Number** of winding planes is **conserved**
- Conserved 2D object = **2-form symmetry**
- **Gauge** U(1) **kills 2-form** symmetry: **mixed anomaly**



Conserved currents

- **Electromagnetic** U(1) **current**

$$\partial_\mu J^\mu = 0, \quad \Longleftrightarrow \quad \partial_t \rho + \boldsymbol{\nabla} \cdot \boldsymbol{J} = 0$$

- **2-form** symmetry has **current** with more indices
- **Anomalous** non-**conservation**

$$\partial_\lambda K^{\lambda\mu\nu} = \tilde{F}^{\mu\nu}$$

Three-form current

- Three-form current **conserves winding-planes**
- Related to **Goldstone**

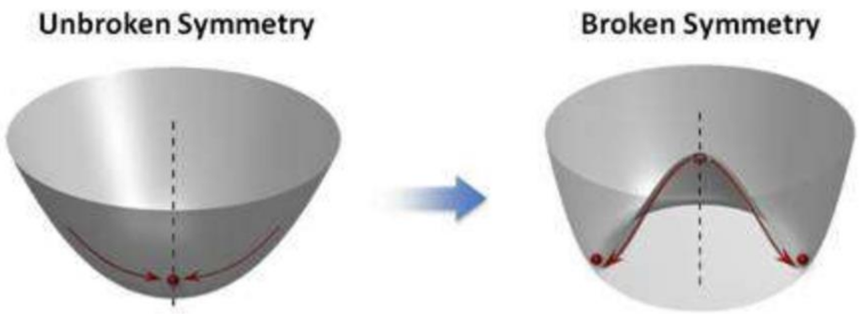
$$K^{\lambda\mu\nu} = \epsilon^{\lambda\mu\nu\rho}\partial_\rho\phi,$$

- **Gauging** $U(1)$ produces **anomaly**

$$K^{\lambda\mu\nu} = \epsilon^{\lambda\mu\nu\rho}(A_\rho + \partial_\rho\phi), \quad \partial_\lambda K^{\lambda\mu\nu} = \tilde{F}^{\mu\nu}$$

Goldstone's theorem

- **SSB** ⇒ shift-symmetric scalar ϕ



- New theorem: Mixed anomaly

$$\partial_\lambda K^{\lambda\mu\nu} = \tilde{F}^{\mu\nu}$$

⇒ shift-symmetric scalar ϕ

Superfluids as Higher-form Anomalies

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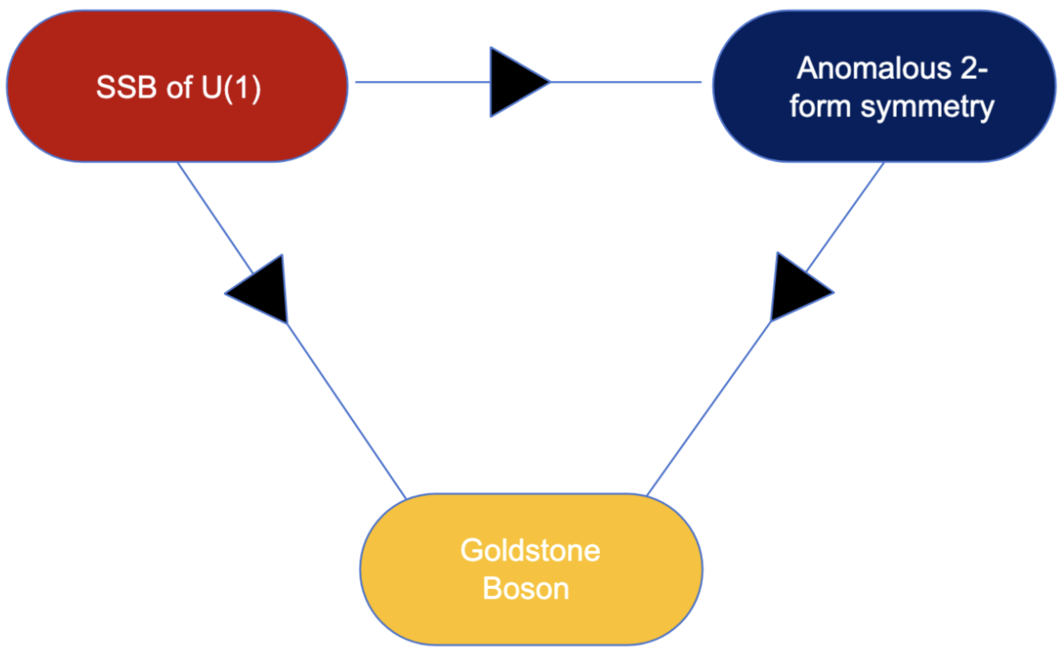
Abstract

We recast superfluid hydrodynamics as the hydrodynamic theory of a system with an emergent anomalous higher-form symmetry. The higher-form charge counts the winding planes of the superfluid – its constitutive relation replaces the Josephson relation of conventional superfluid hydrodynamics. This formulation puts all hydrodynamic equations on equal footing. The anomalous Ward identity can be used as an alternative starting point to prove the existence of a Goldstone boson, without reference to spontaneous symmetry breaking. This provides an alternative characterization of Landau phase transitions in terms of higher-form symmetries and their anomalies instead of how the symmetries are realized. This treatment is more general and, in particular, includes the case of BKT transitions. As an application of this formalism we construct the hydrodynamic theories of conventional (0-form) and 1-form superfluids.

SSB vs Mixed anomaly

- **SSB ⊂ Mixed anomaly**
- Both **yield Goldstone** bosons



Broken vs Unbroken

- **Broken** phase exhibits **full anomaly**

$$\partial_\lambda K^{\lambda\mu\nu} = \tilde{F}^{\mu\nu}$$

- **Unbroken** phase: violation of anomaly

$$\partial_\lambda K^{\lambda\mu\nu} = \tilde{F}^{\mu\nu} + \Gamma^{\mu\nu}$$

- **Moral** of the story: **vortex** $\Gamma^{\mu\nu}$ proliferation **destroys superconductivity**
- Idea: can **violate certain components** but not others

Anomaly structures

$$\partial_\lambda K^{\lambda\mu\nu} = \tilde{F}^{\mu\nu} + \Gamma^{\mu\nu}$$

- **Destroy some components** of anomaly
- **Preserve other** components
- Vortex **density:** Γ^{0i}
- Vortex **current:** Γ^{ij}

	Global $U(1)$ ($F_{\mu\nu} = 0$)	Local $U(1)$ (dynamical $F_{\mu\nu}$)
$\Gamma^{0i} = 0$, $\Gamma^{ij} = 0$	Superfluid	Superconductor
$\Gamma^{0i} \neq 0$, $\Gamma^{ij} \neq 0$	Unbroken phase (e.g. fluid)	Ordinary conductor/insulator
$\Gamma^{0i} \neq 0$, $\Gamma^{ij} = 0$	Superfluid vortex glass	Superconducting vortex glass
$\Gamma^{0i} = 0$, $\Gamma^{ij} \neq 0$	Chimeric superfluid	Chimeric conductor/insulator

EFT approach

- **Conservation** law = **Gauge** invariance of generating functional
- **Gauge** invariance = **Stückelberg** trick
- Modify Stückelberg trick to **account** for **anomaly**
- **Destroy** exact conservation with **non-Stückelberg trick**

Superconductor generating functional

- **Conservation** laws

$$\partial_\mu J^\mu = 0 \qquad \partial_\lambda K^{\lambda\mu\nu} = \tilde{F}^{\mu\nu}$$

- **Generating functional** depends on gauge field

$$W[A_\mu, h_{\lambda\mu\nu}]$$

- **Currents** defined by

$$\hat{J}^\mu(x) = \frac{\delta W}{\delta A_\mu(x)}, \quad \hat{K}^{\lambda\mu\nu}(x) = \frac{1}{3!}\frac{\delta W}{\delta h_{\lambda\mu\nu}(x)}$$

- **Anomalous gauge symmetry** ensures anomalous conservation

$$W\left[A_\mu + \partial_\mu\Lambda, h_{\lambda\mu\nu} + \partial_{[\lambda}\kappa_{\mu\nu]}\right] = W[A_\mu, h_{\lambda\mu\nu}] + \frac{1}{2}\int d^4x\, \epsilon^{\mu\nu\lambda\rho}\partial_\mu A_\nu \kappa_{\lambda\rho}.$$

Stückelberg trick

- To ensure gauge symmetry, **promote anomalous gauge parameters to fields**

$$S = S_0[F_{\mu\nu}, H_{\lambda\mu\nu}] + \int d^4x\, J^\mu A_\mu$$

$$H_{\lambda\mu\nu} = h_{\lambda\mu\nu} + \partial_{[\lambda} b_{\mu\nu]}$$

- **Superconductor** source-free action

$$S_{\mathrm{sc}} = \int d^4x \left(\frac{1}{2}\epsilon \boldsymbol{E}^2 - \frac{1}{2\mu}\boldsymbol{B}^2 + \frac{1}{2\chi}(J^0)^2 - \frac{1}{2}\mu\lambda^2 \boldsymbol{J}^2 + J^\mu A_\mu \right) \quad (20)$$

- **London** EoM

$$\boldsymbol{E} = \mu\lambda^2 \partial_t \boldsymbol{J}, \quad \boldsymbol{B} = -\mu\lambda^2 \boldsymbol{\nabla} \times \boldsymbol{J}$$

Chimeric anomaly

- **No** vortex **density:**

$$\partial_\mu K^{\mu 0 i} = \tilde{F}^{0i}$$

- Vortex **current proliferation:**

$$\partial_\mu K^{\mu ij} = \tilde{F}^{ij} + \Gamma^{ij}$$

- **Interpolates** between **broken** and **unbroken** phases

Non-Stückelberg trick

- Want to **kill** anomalous **conservation** of certain components of $K^{\lambda\mu\nu}$
- Must **partially break gauge invariance**
- Allow $c_i = \frac{1}{2}\epsilon_{ijk}b_{jk}$ to appear outside Stückelberg package
- Still require $b_i = b_{0i}$ to appear in Stückelberg package
- On Schwinger-Keldysh contour, find additional terms:

$$I^{(\mathrm{non})} = \int d^4x \;\; \sigma^{-1}\boldsymbol{c}_a \cdot (iT_0\boldsymbol{c}_a - \partial_t \boldsymbol{c})$$

Dissipative vs dissipationless response

- **Two components** for electric **current**

$$\boldsymbol{J} = \boldsymbol{J}_{\text{cond}} + \boldsymbol{J}_{\text{Meis}}$$

- **Dissipative** response $\boldsymbol{J}_{\text{cond}} = -\partial_t \boldsymbol{c}$
- **Dissipatinoless** response $\boldsymbol{J}_{\text{Meis}} = \boldsymbol{\nabla} \times \boldsymbol{b}$

Chimeric conductor

- **Ohm's law (modified)**

$$\boldsymbol{J}_{\mathrm{cond}} = \sigma \boldsymbol{E}$$

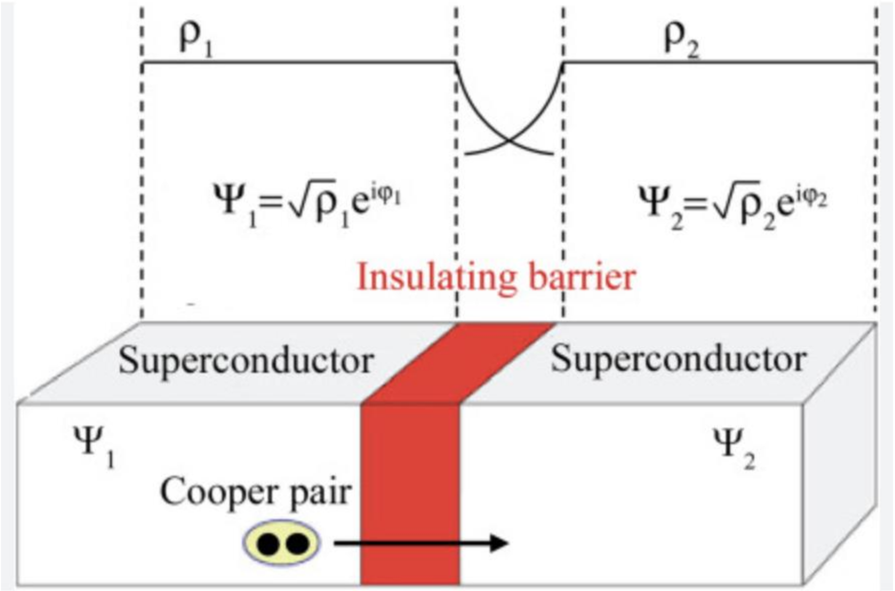
- **Second London** equation

$$\boldsymbol{B} = -\mu\lambda^2 \boldsymbol{\nabla} \times \boldsymbol{J}$$

Lattice model

- Want: **lattice** model for **chimeric** insulators
- Idea: **Josephson junction** network
- Need **something special** to realize chimeric phase

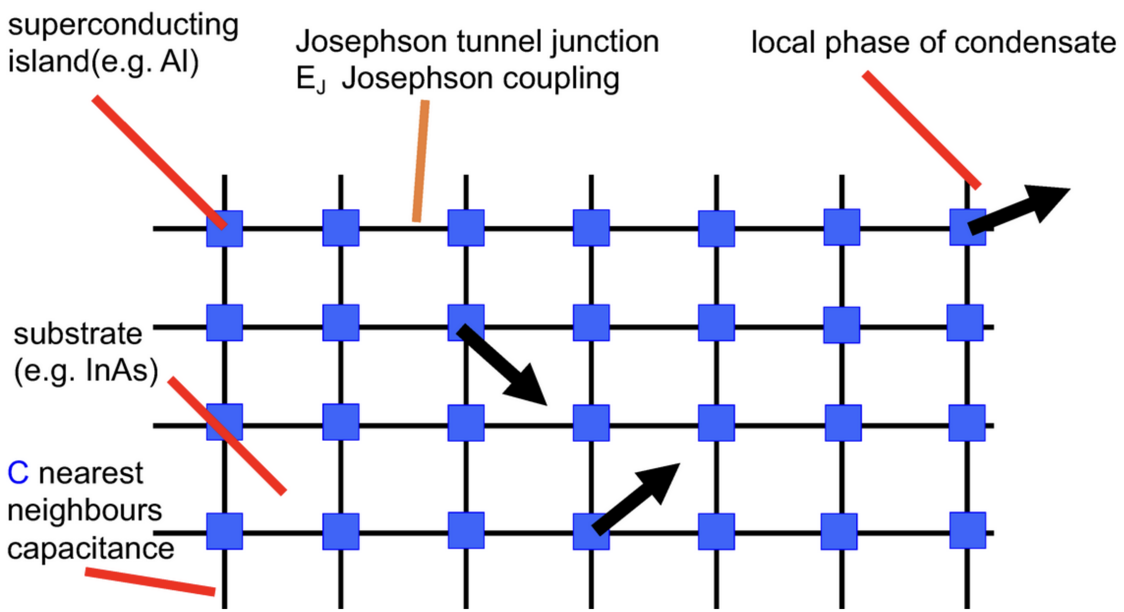
Josephson junction



- Two **superconductors separated** by thin insulator
- **Tunneling** establishes phase coherence
- **Global** superconducting **phase**

Josephson junction network

- **Superconducting islands** connected by Josephson junctions
- Two phases: **superconductor** and **insulator**
- **Superconductor:** Josephson current establishes **global** phase **coherence**
- **Insulator:** Josephson current too weak, **no** global phase **coherence**



Lattice model for chimeric insulator

- Usual **Josephson potential:**

$$- E_J \sum_{\langle ij \rangle} \cos\left(\theta_i - \theta_j - \mathcal{A}_{ij}\right)$$

- **Phase disruptor** field ξ_{ij}
- **Modified** Josephson **potential**

$$- E_J \sum_{\langle ij \rangle} \cos\left(\theta_i - \theta_j - \mathcal{A}_{ij} - \xi_{ij}\right)$$

- Force **curl-free** condition: confining phase for ξ_{ij}
- **Chimeric phase established**

Conclusion

- More possibilities beyond **broken** and **unbroken** phases
- Middleground exists: **chimeric matter**
- Key features of both broken and unbroken states **"shouldn't" coexist**
- Macro theory: generalized symmetry
- Applications: **contactless transport, magnetic shielding, levitation**