New Horizon Symmetries, Hydrodynamics and Chaos

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Introduction

Asymptotic symmetries

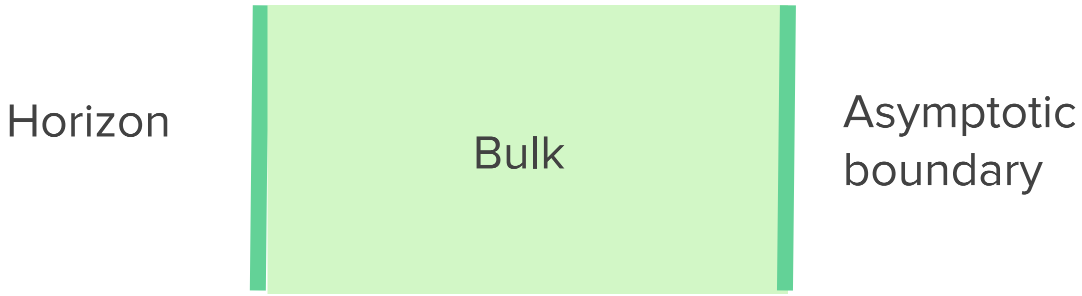
- Asymptotic symmetries are local symmetries that **do not vanish at the asymptotic boundary** of a spacetime and **preserve some boundary conditions**
    - BMS group in flat spacetime: [Bondi, Van der Burg, and Metzner 1962, Sachs 1962]
      Poincare’ group + supertranslations
    - Conformal symmetry group in AdS [Brown, Hennaux 1986]
    - Large gauge transformations in QED [Strominger et al 2014]

Asymptotic symmetries

- Related to physical phenomena (Weinberg's soft graviton theorem, soft photon theorem [see, e.g., Strominger 2017], Displacement Memory effect,...)
- May be interpreted as the **symmetries of the dual boundary system**
  - AdS/CFT correspondence [Maldacena 1998]
  - Celestial (flat) holography [Pasterski, Shao, Strominger, 2017]
- Challenges ahead
  - *Which boundary conditions are physically correct?*
  - ....

Horizon symmetries

- Horizon symmetries are local symmetries that **do not vanish** at the **horizon** of a **black hole** and **preserve some boundary conditions**
    - Stationary **black holes** possess an **infinite amount of symmetries** beyond the usual Killing symmetries [Carlip 1999, Hotta et al. 2001, i.-Koga 2001, Guica et al. 2009, Compere et al 2012] [Donnay et al. 2015 + ...]
    - Symmetries of **any null surface** and can be characterized **covariantly** [Chandrasekaran et al 2018]
    - ***Interpretation** from the dual boundary point of view (even in the familiar AdS case) **is unclear*** [Eling and Oz 2016, Penna 2017, Donnay and Marteau 2017, Marjieh, **NPF**, Tavor and Yarom 2021]



Results:

Interpretation:

Horizon symmetries are symmetries of the **dual hydrodynamic** and **chaotic boundary theory**

Results:

1. New horizon symmetries

2. **Interpretation:**

Horizon symmetries are symmetries of the **dual hydrodynamic** and **chaotic boundary theory**

3. Implications on shock waves and pole skipping

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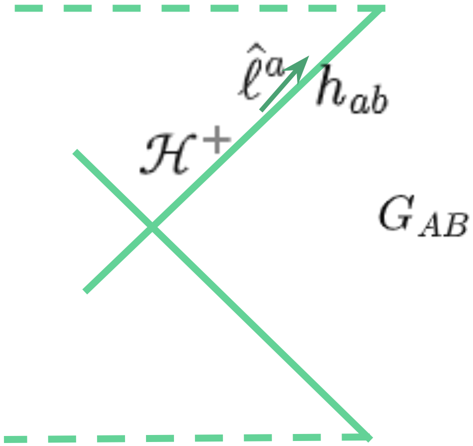
Null surfaces

- Consider a null manifold with coordinates x^A and metric G_{AB}
- Given the embedding functions $X^A(\sigma)$, the induced metric on the horizon is

$$h_{ab}(\sigma) = \partial_a X^A(\sigma)\partial_b X^B(\sigma)G_{AB}(X(\sigma))$$

- The metric h_{ab} is degenerate: $h_{ab}\hat{\ell}^a = 0$
- The null vector: $\ell^A \triangleq \partial_a X^A \hat{\ell}^a, \quad \ell^A \ell^B G_{AB} \triangleq 0$
- Non-affinity parameter: $\kappa \ell^A \triangleq \ell^B \nabla_B \ell^A$



Horizon symmetries

- Consider bulk diffs: $x^A \to x^A + \chi^A, \quad G'_{AB} = G_{AB} + \pounds_\chi G_{AB}$
- They imply: $h'_{ab} = h_{ab} + \partial_a X^A \partial_b X^B \pounds_\chi G_{AB}(X)$

$$\hat{\ell}'^{\,a} = \hat{\ell}^a, \quad (\ell'^{\,A} = \ell^A), \quad \kappa' \ell^A \triangleq \ell^B \nabla'_B \ell^A$$

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  $\hat{\ell}'^a = \hat{\ell}^a, \quad (\ell'^A = \ell^A), \quad \kappa' \ell^A \triangleq \ell^B \nabla'_B \ell^A$

- χ^A is a **horizon preserving diffeomorphism** if

  i) the null manifold remains null $h'_{ab}\hat{\ell}^a = 0$

  ii) the non-affinity parameter is unchanged $\kappa' = \kappa$

→ **Horizon symmetry constraints**

i) $\pounds_\chi G_{AB} \ell^B \triangleq 0$

ii) $\nabla_C (\pounds_\chi G_{AB} \ell^A \ell^B) \triangleq 0$

Horizon symmetries: Example

- Consider diffs along the horizon: $\chi^A \ell_A \triangleq 0$
- For horizons parameterized by

$$ds^2_{\mathcal{H}} = h_{ab} d\sigma^a d\sigma^b = \gamma_{ij} d\sigma^i d\sigma^j \qquad \hat{\ell}^a = \left(\frac{\partial}{\partial \sigma^0}\right)^a$$

- The horizon preserving diffs can be decomposed into **supertranslations** and **superrotations**

$$\sigma^0 \to \sigma^0 + f(\sigma^0, \vec{\sigma}), \qquad \vec{\sigma} \to \vec{\sigma} + \vec{Y}(\sigma^0, \vec{\sigma})$$

- which satisfy the constraints $\partial_0 Y^i = 0\,, \qquad \partial_0^2 f + \kappa \partial_0 f = 0$
- and leads to

$$\sigma^0 \to \sigma^0 + \lambda(\vec{\sigma}) + \alpha(\vec{\sigma}) e^{-\kappa \sigma^0}$$
$$\sigma^i \to \sigma^i + \zeta^i(\vec{\sigma})$$

[Donnay et al. 2015 + ...]

New horizon symmetries

- Consider diffs not aligned on the horizon $\chi^A \ell_A \neq 0$
- The horizon symmetry constraints imply

Shift symmetries: postulated as symmetries of **chaotic systems*** [Liu, Lee, Blake 2018]

$$\sigma^0 \to \sigma^0 + \lambda(\vec{\sigma}) + \alpha(\vec{\sigma})e^{-\kappa\sigma^0} + \gamma(\vec{\sigma})e^{\kappa\sigma^0}$$

$$\sigma^i \to \sigma^i + \zeta^i(\vec{\sigma}) + \#\partial_i\gamma(\vec{\sigma})e^{\kappa\sigma^0}$$

Hydro symmetries

New prediction

* Maximally chaotic, no momentum conservation

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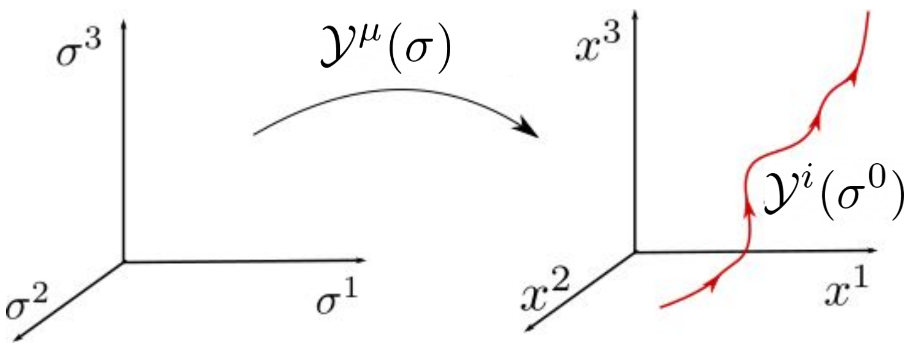
Modern hydrodynamic d.o.f.

- Conventional fluid d.o.f. are: $u^\mu(t,\vec{x}),\ \ T(t,\vec{x}),\ \ \mu(t,\vec{x})$
- Modern fluid d.o.f. are **maps:** $\mathcal{Y}^\mu(\sigma),\ \ \phi(\sigma)$



Fluid (Lagrangian) **frame**

Laboratory (Eulerian) **frame**

- The resulting e.o.m. from a variational principle are the hydrodynamic equations [Herglotz 1911, Taub 1954, Carter 1973,...]

$$0 = \frac{\delta S_{eff}}{\delta \mathcal{Y}^\mu} = \nabla_\nu T^{\mu\nu} \qquad 0 = \frac{\delta S_{eff}}{\delta \phi} = \nabla_\mu J^\mu$$

The redundancy

- Relation to the conventional variables:

$$\frac{\partial \mathcal{Y}^\mu}{\partial \sigma^0} = \beta^\mu = \frac{u^\mu}{T} \qquad \mu = \frac{\partial \phi}{\partial \sigma^0} + \beta^\mu A_\mu$$

- The following transformation leaves the thermal vector and chemical potential invariant:

$$\sigma^0 \to \sigma^0 + \lambda(\vec{\sigma}), \qquad \sigma^i \to \sigma^i + \zeta^i(\vec{\sigma})$$

$$\phi \to \phi + \Lambda(\vec{\sigma})$$

- This local symmetry, together with other properties of S_{eff} based on the Schwinger-Keldysh path integral (CPT, KMS,...), leads to a **consistent EFT for hydrodynamics** [Crossley, Glorioso, Liu 2015-2018, Haehl, Loganayagam, Rangamani 2015-2018, Marjieh, Jensen, **NPF**, Yarom 2017-2018]

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EFT for chaos

- Hydrodynamics in strongly coupled CFTs assumes $L \gg \beta$
- The EFT for hydrodynamics can be extended to orders $L \sim \beta$:

  quantum hydrodynamics = hydro to all order in derivatives
- **Quantum hydrodynamics + shift symmetry**

$$\sigma^0 \to \sigma^0 + \lambda(\vec{\sigma}) + \alpha(\vec{\sigma}) e^{-\kappa\sigma^0} + \gamma(\vec{\sigma}) e^{\kappa\sigma^0}$$

$$\sigma^i \to \sigma^i + \zeta^i(\vec{\sigma})$$

= an **EFT for maximally chaotic systems*** [Liu, Lee, Blake 2018]

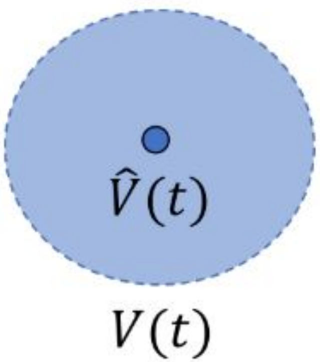
* Maximally chaotic, no momentum conservation

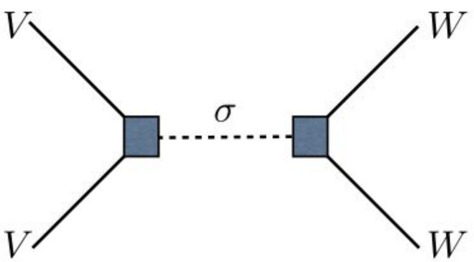
EFT for chaos

- Exponential growth of Out-of-Time Ordered (OTOC) correlation functions (and no growth of TOC) [Shenker, Stanford 2013, Kitaev 2015,...]

$$\langle [V(t), W]^2 \rangle \sim \frac{1}{N} e^{\kappa t}$$

- An EFT for chaos*





  - Exponential growth of the propagator $G_{\sigma^0 \sigma^0} \sim e^{\kappa t}$
  - Pole skipping [Grozdanov, Schalm, Scopelliti 2017, Blake, Davison, Grozdanov, Liu 2018]

$$G_{\epsilon\epsilon} \sim \frac{(\omega - \omega^*)}{k - k^*}$$

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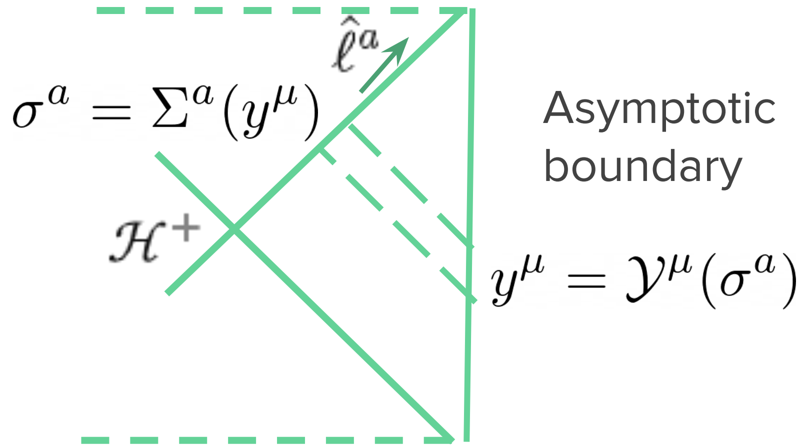
Fluid d.o.f. in gravity

- In gravity, fluid d.o.f. are **maps between the horizon** and the **boundary** [Nickel and Son 2010, Liu, Glorioso et al 2015, de Boer, Heller, **NPF** 2015]
(See, e.g., the time reparameterization mode in JT gravity [Jensen 2016])
- Shoot a (null) geodesic from the point y^μ on the boundary to the point σ^a on the horizon. This establishes a map $\sigma^a = \Sigma^a(y^\mu)$



- The velocity field is given by: $\beta^\mu = \dfrac{\partial \mathcal{Y}^\mu}{\partial \sigma^a}\hat{\ell}^a$

Fluid d.o.f. in gravity

- Rewrite the metric as follows

$$ds^2 = -2a_\mu dx^\mu dr + \chi_{\mu\nu}(r,x)dx^\mu dx^\nu$$

- Null geodesics correspond to $x^\mu = const.$ and the tangent vector is
  [Bhattacharyya, Hubeny, Loganayagam, Mandal, Minwalla, Morita, Rangamani, Reall 2008]

$$T^A = (1, a^\mu(y))$$

- Starting from the boundary (r_B, y^μ) the horizon is reached at $(r_\mathcal{H}, y^\mu)$
- And the map is then given by $\mathcal{Y}^\mu(\sigma) = x^\mu(\sigma)|_\mathcal{H} = X^\mu(\sigma)$
- which transforms under the horizon symmetries as

$$\mathcal{Y}^\mu \to \mathcal{Y}^\mu + \chi^\mu$$

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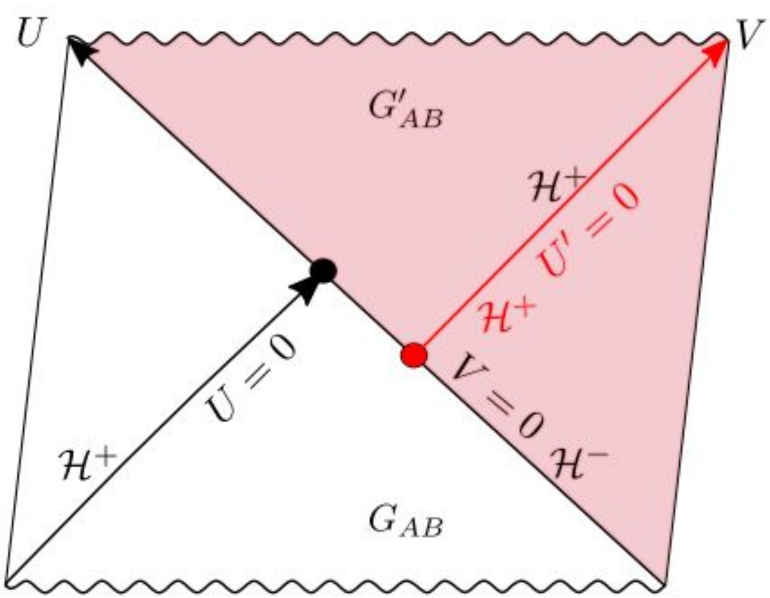
Shock Waves

The new horizon symmetries can be used to create shock wave geometries relevant for OTOCs

- Shock wave geometry generated by a high energy particle injected at V=0
  [Sfetsos 1995]

$$G_{\text{shock}} = G + \Theta(V)\mathcal{L}_\xi G\,, \qquad \xi^V = 0\,, \quad \xi^U = q(\vec{x})\,, \quad \xi^i = 0$$



Shift symmetry and shock waves

- Shock wave geometry generated by the horizon symmetry transformation

$$G'_{\text{shock}} = G + \Theta(V)\mathcal{L}_\chi G\,,$$

$$\chi^V = \#\gamma(\vec{x})V^2 + \mathcal{O}(U)\,,$$
$$\chi^U = \#\gamma(\vec{x}) + \mathcal{O}(U)\,,$$
$$\chi^i = \#\partial_i\gamma(\vec{x})V + \mathcal{O}(U)$$

- is related to the original shock wave geometry along U=0 via a diffeo

$$G_{\text{shock}} = G'_{\text{shock}} + \mathcal{L}_\rho G'_{\text{shock}}\,, \qquad \rho = -\Theta(V)(\chi - \xi)$$

- Thus, the two seemingly different geometries are physically equivalent

Chaos pole-skipping point

- The 2-point function for the energy density correlator exhibits the pole-skipping phenomenon: a would-be pole appears to be skipped at: (ω^*, k^*)

$$\omega^* = i\kappa\,, \quad k^{*2} = -d\pi T g'(r_{\mathcal{H}})$$

- The value of ω^* is universal and determined by the horizon symmetry

- The value of k^* is model dependent. For instance, it is determined by the shock wave equation

$$\left(\nabla^2 - d\pi T g'(r_{\mathcal{H}})\right) q(\vec{x}) \sim \delta(\vec{x})$$

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Outlook

- Generalizations to
  - charged black holes, non stationary black holes, extremal black holes,...
  - higher derivative and stringy corrections (non maximal chaos),
  - non asymptotically AdS spacetimes (Rindler fluids) ...
- Is the new prediction helpful to constrain a complete EFT for maximal chaos?
- Gravitational dual of the symmetry responsible for entropy production?
- Is there a symmetry responsible for the infinite tower of pole skipping points?
- Relation to edge modes?

Thank you!

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Extra slides:

The equations of motion

For example, to write a Lorentz covariant $T^{\mu\nu}$ it is convenient to couple the theory to an external source $g_{\mu\nu}$ and require diff invariance

The pullback sources: $$g_{ij} = \frac{\partial X^\mu}{\partial \sigma^i} \frac{\partial X^\nu}{\partial \sigma^j} g_{\mu\nu}(X(\sigma))$$

Sigma-model Lagrangian: $$S_{eff} = \int d^d\sigma \sqrt{-g} \mathcal{L}(g_{ij})$$

The constitutive relations: $$T^{ij} = \frac{2}{\sqrt{-g}} \frac{\delta S_{eff}}{\delta g_{ij}}$$

E.o.m.: $$0 = \delta_X S_{eff} = \int d^d\sigma \sqrt{-g} (\nabla_i T^{ij}) \partial_j X^\mu g_{\mu\nu} \delta X^\nu$$

Fluid d.o.f. in gravity: e.o.m.

- The action for the new metric is

$$S[G_{AB}] = S[\tilde{G}_{AB}, \tilde{x}^A]$$

- The variation of the action

$$\delta S = \frac{1}{2}\int d^D x \sqrt{-G} E^{MN} \delta G_{MN}$$

- Variation w.r.t. the dynamical d.o.f.

$$0 = \frac{\delta S}{\delta \tilde{x}^{\mu}} \longrightarrow E^{\mu r} = 0 \longrightarrow \nabla_{\nu} T^{\mu\nu} = 0$$

Fluid d.o.f. in gravity: example

- Consider linearized perturbations on a black hole background: $G_{AB} + f_{AB}$
- To go to the desired parameterization of the metric we perform a diffeomorphism and require $f_{Ar} + \pounds_\xi G_{rA} = 0$
- The diffeomorphism evaluated on one of the boundaries gives the linearized fluid d.o.f. $\mathcal{Y}^\mu(\sigma) = \delta^\mu_a \sigma^a + \pi^\mu(\sigma)$

$$\pi^v \sim \int_{\mathcal{H}}^{B} f_{rr} dr' \,, \quad \pi^i \sim \int_{\mathcal{H}}^{B} f_{ri} dr'$$

- Which transform under the horizon symmetries: $\pi^\mu \to \pi^\mu + \chi^\mu(\mathcal{H})$